Stochastic Multi-attribute Decision Making Method Aiming at Effectiveness

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Abstract—A multi-attribute decision making method for the general scheme of weapon equipment which aims at the system effectiveness is proposed in this paper. This method considers the stochastic confidence interval characteristics of reliability and maintainability. The objective set of alternatives is established according to the attributes-based measuring method for effectiveness. The random distance functions and random relative distance functions of alternatives to the positive and negative ideal schemes are established. The simulation arithmetic based on MATLAB for random relative distance function is given, and the alternatives' relative priority possibility degree matrix is determined according to random relative distance function, then the alternatives' priority with quantitative risk is given. Finally, a decision making case about the carrier aircraft general scheme was given to prove accuracy of model.

Keywords- Effectiveness; multi-attribute decision making; Reliability; Maintainability; Carrier Aircraft

I. INTRODUCTION

Effectiveness is the ability of weapon equipment to accomplish a mission, and achieve an expected outcome. It plays importance role to establish the relationship between development and operation of the equipment [1]. Measures of effectiveness (MOE) are outcome-oriented measure sets of system’s effectiveness. These measure maybe probabilistic or physical measures and they allow comparable systems to be analyzed, ranked and chosen [2].

The early models of MOE are relatively simple, and always described in a probabilistic manner. The most influential model of these is ADC model which was proposed by WSEIAC in 1964, where A is availability, D is dependability, C is capability.

With the equipment and its task becoming more and more complex, sinle scale measure was unable to satisfy system’s measuring requirement, and the measure cannot be evaluated through analytic approach also. Now, American Army usually measuring equipment’s effectiveness in terms of system attributes during the development of new weapon equipment [1, 3]. Further, the multi-attribute decision making theory is applied to decision making and choosing of alternatives [4].

Reliability and maintainability (R&M) are the important attributes of weapon equipment, and important parts of system effectiveness [5].

There are many uncertainties exist in evaluation of R&M during equipment development because of difference of R&M historical data or smaller samples. It result that R&M has less influence to scheme, and cannot be adopted as an important factor in decision-making of general scheme during engineering practice. The existing decision making process has not enough description on characteristics of operation and support of equipment and the scheme cannot reflect the effectiveness of equipment completely.

Some researcher applied point estimation of R&M in effectiveness evaluation and decision making of scheme directly, but it brings more risk in decision making process. R&M parameters should be described by confidence interval and participate in decision making process as a risk function. The decision making problem become complex for consideration of confidence interval, and it should be solved by comparison of stochastic areas in decision making space rather than comparison of points.

This type of stochastic multi-attribute decision-making problem cannot be solved well by existing general method (such as AHP method, TOPSIS method, and grey correlation analysis method) for lack of enough theory foundation [6-8]. Some researchers has proposed some methods (such as risk expectation theory and interval decision making) considering the influence of stochastic factors [9-11], but these methods also cannot meet the decision making requirement with confidence interval.

This paper presents basic rules for measuring equipment’s effectiveness based on attributes sets, discusses the definition method of attribute weight, and establish the objective set of alternatives according to MOE. It provides a new decision making method which use confidence interval estimation of R&M under certain confidence level participating in decision making process. Further, it establishes the random distance functions and random relative distance functions of alternatives to the positive and negative ideal schemes according to TOPSIS theory. Then, a simulation arithmetic based on MATLAB for these decision making functions is given to rank alternatives by possibility degree matrix with a quantitative decision making risk. Finally, the decision making model is verified through the case analysis of general scheme decision making of carrier aircraft.

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II. DESCRITIONS OF STOCHASTIC DECISION MAKING PROBLEM AIMING AT EFFECTIVENESS

A. Attributes Selection and Weights Definition

MOE are standards against which the capability of a solution to meet the needs of a problem may be judged. The standards are representative of those properties which any potential solution must exhibit to some extent. MOEs are independent of any solution and do not specify performance or criteria [3]. According to this view, a set of parameters which characterize the ability of equipment performing task can be applied to measure system’s effectiveness. Due to the existence of subjective influence of decision maker in MOE establishing, the reference criterion is needed to given. Among these criterions, the measurability and independency are the basis. The former requires that the parameters can be obtained from mathematic method, physical test or computer simulation. The later requires the relationship among parameters should not be a containing relationship but a nearly mutual exclusion relationship.

The complex equipment is anticipated to perform many tasks or missions. For each task and mission, a set of characteristic parameters can be obtained. When there is too much parameter to justify the key parameter, the AHP method can be referred [6] for selecting parameter with the maximum weight value as measure attribute.

Step 1: Establish the comparison matrix for candidate factors. The five level grading standards {1, 3, 5, 7, 9} is utilized for valuing the matrix. Where \( a_{ij} \) denotes the same importance degree with \( a_{ij} \). Then, the decision making matrix, the weight value for actual decision parameters can be obtained from expression (1) with the maximum weight value as measure attribute.

\[
W_i = \frac{1}{\sum_{j=1}^{n} b_j} \quad (i = 1, 2, \ldots, n)
\]

Step 2: Determine the weights for different factors with the root of matrix.

Calculate the relative weight \( W_i \) and normalized weight \( \omega_i \) with expression (1).

\[
\omega_i = W_i / \sum_{i=1}^{n} W_i
\]

Step 3: Sort the parameters according to calculated weight value. The prior parameter (with large weight value) is selected as the key capability parameter. Then, by solving the reduced decision making matrix, the weight value for actual decision parameters can be obtained.

The objective set for general scheme decision making of weapon equipment aiming at effectiveness \( U \) is described as \( [u_{11}^c, u_{12}^c, \ldots, u_{21}^c, u_{22}^c] \). The parameters in \( U \) can be classified into two types, performance index and R&M index. Performance index can be evaluated accurately. However, there is still uncertainty exist in these parameters due to un-finalization of the design during scheme decision making. For optical selection of schemes, the interval numbers can be used to describe the performance index of equipment. On the other hand, R&M index are regarded as independent stochastic attribute (confidence interval) in decision making.

The weight set for each factor is denoted by \( \omega = (\omega_1, \omega_2, \ldots, \omega_n) \), where \( \omega \) can be obtained from expression (1) with subjective weighting method. It also can be obtained from information of decision making matrix with objective weighting method.

B. Definition of Decision Making Matrix and Ideal Schemes

Combined with the characteristic of parameters, the decision making matrix \( D \) is established during general scheme decision making of equipment. Let \( X = \{x_1, x_2, \ldots, x_m\} \) be the candidate scheme sets for general design of equipment, the assessment value for scheme \( x_i \) to attribute \( u_j \) is denoted by \( [r_{ij}^v, r_{ij}^c] \). Then, the decision making matrix \( D \) for general scheme of equipment is denoted as:

\[
D = [x_1, x_2, \ldots, x_m, u_1, u_2, \ldots, u_n] = [r_{ij}^v, r_{ij}^c]
\]

In this matrix, \( [r_{ij}^v, r_{ij}^c] \) \( j=1, 2, \ldots, n \) is the interval estimation value of attribute \( u_i \), while \( [r_{ij}^v, r_{ij}^c] \) \( j=n+1, n \) is confidence interval estimation value of MOE attribute, which has a certain distribution, under a certain confidence level.

Two definitions are given to further describe the problem of weapon equipment general scheme decision making aiming at effectiveness.

Definition 1: Scheme decision space \( Z \). It is a multi-dimensional vector space composed of coordinate axis of attributes (such as the maximum speed of a plane \( \mathcal{V} \) and fight radius \( \mathcal{S} \)) In the decision space, the position of decision scheme is determined by scheme attributes.

Definition 2: Positive (negative) ideal decision scheme \( x_0^+ (x_0^-) \). It is a virtual scheme composed of relative optimal (worst) values of attribute in each scheme, and its attributes reach the optimal (worst) values of all schemes.

III. DECISION MAKING METHOD BY CONSIDERING ATTRIBUTE CONFIDENCE INTERVAL FEATURES

Step 1: Normalize attribute values in decision matrix \( D \) which aiming at effectiveness to obtain a normalization decision matrix \( B \) [12]:

\[
B = [b_i], i=1, 2, \ldots, m \quad j=1, 2, \ldots, n
\]

Normalize benefit attributes with expression 2, and normalize cost attributes with expression 3.

\[
\begin{align*}
\bar{b}_i^v &= r_{ij}^v / \sqrt{\sum_{j=1}^{n} (r_{ij}^v)^2} \\
\bar{b}_i^c &= r_{ij}^c / \sqrt{\sum_{j=1}^{n} (r_{ij}^c)^2}
\end{align*}
\]

Step 2: Calculate the relative weight \( W_i \) and normalized weight \( \omega_i \) with expression (1).

\[
\omega_i = W_i / \sum_{i=1}^{n} W_i
\]

Step 3: Sort the parameters according to calculated weight value. The prior parameter (with large weight value) is selected as the key capability parameter. Then, by solving the reduced decision making matrix, the weight value for actual decision parameters can be obtained.
\[
\begin{align*}
\hat{y}_j^i &= \frac{(1/\hat{c}_j^i)}{\sqrt{\sum_{m=1}^n (1/\hat{c}_j^i)^2}} \\
\hat{y}_i^j &= \frac{(1/\hat{c}_j^i)}{\sqrt{\sum_{m=1}^n (1/\hat{c}_j^i)^2}} 
\end{align*}
\]

\( i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n \) \hspace{1cm} (3)

Step 2: Determine weighted decision matrix \( B' \):

\[ B' = [\omega_i B_j], j = 1, 2, \ldots, m \text{ and } i = 1, 2, \ldots, n \]

Step 3: Determine positive and negative ideal scheme \( x_0^+ \) and \( x_0^- \) of equipment general scheme aiming at effectiveness:

Where, performance attributes are described by intervals, and the weighted optimal (worst) interval can be regarded as \( x_0^+ \) (\( x_0^- \)).

Reliability and maintainability attributes are confidence interval estimation value under a certain distribution. To convenient description of R&M in scheme space \( Z \), they are considered as random point in certain interval. After normalizing their distributions, 0 and 1 can be used to describe their optimal values and worst values separately. Then the positive ideal scheme \( x_0^+ \) is shown in expression 4:

\[ \left\{ \begin{array}{l}
\max \omega [b_{ij}^+, b_{ij}^-], j = 1, 2, \ldots, n - 2, \omega_{m1} \times 1, \omega_{m2} \times 1 \end{array} \right. \]  

(4)

The negative ideal scheme \( x_0^- \) is shown in expression 5:

\[ \left\{ \begin{array}{l}
\min \omega [b_{ij}^+, b_{ij}^-], j = 1, 2, \ldots, n - 2, \omega_{m1} \times 0, \omega_{m2} \times 0 \end{array} \right. \]  

(5)

Step 4: Calculate each alternative distance function \( L^+_i \) and \( L^-_i \) from each alternative \( x_i \) to the positive ideal scheme \( x_0^+ \) and the negative ideal scheme \( x_0^- \).

For alternative \( x_i \) use stochastic attributes \( y_{m1}^{x_i}, y_{m2}^{x_i} \) to express R&M attributes values in scheme, then \( L^+_i \) and \( L^-_i \) can be calculated as expression 6 and 7:

\[
L^+_i = \sum_{j=1}^{n-2} \omega_j [b_{ij}^+, b_{ij}^-] - \max_{j=1}^{n-2} \omega_j [b_{ij}^+, b_{ij}^-] \right) \right)^2 \\
L^-_i = \sum_{j=1}^{n-2} \omega_j [b_{ij}^+, b_{ij}^-] - \min_{j=1}^{n-2} \omega_j [b_{ij}^+, b_{ij}^-] \right) \right)^2 \\
+ (\omega_{m1} y_{m1}^{x_i} - \omega_{m2} y_{m2}^{x_i})^2 \\
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by relative possibility degree, which considered stochastic decision risk.

IV. CASE ANALYSIS

Carrier aircraft is a kind of weapon equipment which takes aircraft carrier or large ship as boarded platform [13]. It was used to implement many tasks and missions, such as antiair warfare, fire against shore and patrol. Speed and the combat distance are two classes of key parameters which can described fire against shore mission and the vigilance patrol mission (to guard aircraft carrier or demonstrate). These two classes of key parameters can be measured by maximum speed ($V$) and fight radius($S$) respectively. On the other hand, antiair warfare mission requires the carrier aircraft to be act quickly and flexibly. So it can be measured by maneuverability ($L$). Further, maneuverability can be decomposed into many performances such as sustained hover performance, level acceleration performance, aerobat performance and maneuvering performance. Based on these parameters, the weighted average method could be used to evaluate the maneuverability of the carrier aircraft [14]. Besides, the attack ability required by three missions is closely related to bomb load, and it can be measured by maximum takeoff weight ($W$) after combining with fuel quantity (have important influence to fight radius).

Furthermore, R&M parameters are used to characterize the capacity that tasks can be performed successfully. Mean fly hours between failures (MFHBF) and direct maintenance man hours per fly hour (DMMH/FH) which are closely related to carrier aircraft operation are chosen in this case.

Then objective set $U = \{u^1_1, u^2_1, u^3_1, u^4_1, u^5_1, u^6_1\}$ , it is composed of maximum speed, fight radius, maximum takeoff weight, maneuverability, reliability and maintainability.

Determine weight of weight of attribute according to combination of decision maker preference and expert opinions, then $\omega = [0.2, 0.2, 0.2, 0.1, 0.2, 0.1]$.

Three candidate schemes $x_1, x_2, x_3$ are assumed and their attribute sets are listed in table1. In table, maximum speed (Mach number), fight radius (km), maximum takeoff weight (ton), and maneuverability are described by intervals directly. While R&M are described by confidence interval estimation under certain confidence level $1-\alpha=0.975$(they obey normal distribution).

### TABLE I. ALTERNATIVE DESIGN SCHEMES OF CARRIER AIRCRAFT

<table>
<thead>
<tr>
<th>ATTRIBUTE</th>
<th>SCHEME</th>
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<tbody>
<tr>
<td>maximum speed ($u_1$)</td>
<td>$[1.8, 2]$</td>
<td>$[2.2, 2]$</td>
<td>$[1.7, 1.9]$</td>
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<tr>
<td>fight radius ($u_2$)</td>
<td>$[780, 910]$</td>
<td>$[1150, 1270]$</td>
<td>$[1033, 1853]$</td>
</tr>
<tr>
<td>maximum takeoff weight ($u_3$)</td>
<td>$[29, 30]$</td>
<td>$[25, 27]$</td>
<td>$[21, 23]$</td>
</tr>
<tr>
<td>maneuverability ($u_4$)</td>
<td>$[0.80, 0.85]$</td>
<td>$[0.85, 0.95]$</td>
<td>$[0.86, 0.92]$</td>
</tr>
<tr>
<td>reliability ($u_5$)</td>
<td>$[4.5, 5.5]$</td>
<td>$[3.5, 4.5]$</td>
<td>$[4.5]$</td>
</tr>
<tr>
<td>maintainability ($u_6$)</td>
<td>$[7.5, 8.5]$</td>
<td>$[6.5, 7.5]$</td>
<td>$[7.8]$</td>
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</table>

Decision making is performed by the mentioned model according to following steps:

Step 1:Standardize the decision making matrix in table1 according to expression 1 and 2, and establish weighted standardization matrix $B'$ combining weights.

Step 2: Determine positive and negative ideal decision schemes $x^+_0, x^-_0$ in the scheme space $Z$ according to expression 3 and expression 4:

$$
\begin{bmatrix}
0.102, 0.126 & 0.032, 0.053 & 0.094, 0.103 & 0.051, 0.059 & 0.078, 0.118 & 0.047, 0.061 \\
0.113, 0.138 & 0.047, 0.073 & 0.081, 0.093 & 0.054, 0.066 & 0.060, 0.097 & 0.054, 0.071 \\
0.096, 0.119 & 0.043, 0.107 & 0.068, 0.079 & 0.055, 0.063 & 0.069, 0.108 & 0.050, 0.066 \\
\end{bmatrix}
\begin{bmatrix}
0.113, 0.138 & 0.094, 0.214 & 0.125, 0.137 & 0.055, 0.066 & 0.2 & 0.1 \\
0.096, 0.119 & 0.064, 0.106 & 0.091, 0.105 & 0.051, 0.059 & 0.2 & 0.1 \\
\end{bmatrix}
$$
Step 3: Establish decision making model aiming at effectiveness. Compute the possibility degree $P_{ij}$ of pairwise compare the relative distance $z_i$ between each scheme and positive and negative ideal schemes $x_0^+, x_0^-$ in scheme space $Z$ based on MATLAB platform, then establish possibility degree matrix $P$:

$$P = \begin{bmatrix}
0.512 & 0.305 \\
0.389 & 0.611 \\
- & 0.488 & 0.611 \\
\end{bmatrix}$$

Step 4: Rank of the schemes according to possibility degree matrix $P$ is $x_2 \succ x_1 \succ x_3$. The risk description is scheme $x_2$ has precedence over scheme $x_1$ in probability 0.512; scheme $x_1$ has precedence over scheme $x_3$ in probability 0.611. Combined with the attribute weights in this case, the precedence rank get by the decision method in references [9-11] is also $x_2 \succ x_1 \succ x_3$. The method proposed in this paper is more superior for considering quantitative risk of decision making.

V. CONCLUSION

The necessity and the technical problems of weapon equipment general scheme decision making aiming at effectiveness are discussed in this paper. Combine with the statistical characteristics of R&M attributes (key attributes affecting effectiveness), the solutions of decision making with R&M are presented and verified. The method proposed in this paper is an extended application of stochastic multi-attribute decision making methods; it can be used to support multi-attributes trade-off and decision making of alternatives for development of weapon equipment aiming at effectiveness. The main conclusions are as follows:

1. The basic method and process of weapon equipment general scheme stochastic multi-attribute decision making aiming at effectiveness is proposed in this paper. The method presented in this paper can be used to describe the statistical characteristic of R&M parameters. And it can also describe the quantitative risk of decision making as well as increase creditability of the decision making.

2. The proposed decision making method was proved to be feasibility through the case study in this paper.

REFERENCES