Failure Rate Model of Components with the Number of Load Application as Life Parameter

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Abstract— The failure rate model of components with the number of load application as life parameter is developed in this paper. Taking static strength failure and fatigue failure as the backgrounds, the dynamic reliability models of components under random repeated load without strength degradation and those with strength degradation are derived, respectively. Then, the failure rate model of components with the number of load application as life parameter is derived without strength degradation and those with strength degradation are derived, respectively. The relationship between reliability and the number of load application, and that between failure rate and the number of load application are discussed in different cases. The result shows that even though when strength doesn’t degenerate, both reliability and failure rate of components decrease as the number of load application increases, and the failure rate curve has the partial character of bathtub curve with early failure period and random failure period. When strength degenerates with the number of load application, the reliability of components decreases more obviously, and the failure rate curve of components is bathtub-shaped.

Keywords- failure rate; reliability model; the number of load application; load-strength interference

I. INTRODUCTION

The existing failure rate models are always derived from the life distribution function which is developed through the statistical analysis of fail or test data. However, these models hardly embody the relationship between the failure rate and parameters, such as load and strength et al [1]. At the same time, the reliability and failure rate of components or systems is always expressed as the function of time. In fact, there are a lot of mechanical and electronic products, whose life is measured by the number of load application. For example, the number of load application is used to measure the life of bearings subjected to the cyclic period stress, the number of charge or discharge is applied to measure the life of impulse capacitors, and so on. In order to analyze and evaluate conveniently the reliability of products, whose life parameter is denoted by the number of load application, it is necessary to develop the reliability and failure rate models of components and systems when the number of load application is taken as the life parameter.

In this paper, the failure mechanism of components under repeated random load application is analyzed, and the reliability and failure rate models of components under random repeated load are derived. Then, the relationship between the reliability and the number of load application, and that between the failure rate and the times of load application will be discussed in different case.

II. RELIABILITY MODEL OF COMPONENTS UNDER REPEATED RANDOM LOAD

Random loads which are applied to components or systems in the service life are almost always repetitious, and the effect of the number of load application on reliability should be considered. However, most of reliability models were derived directly through the load-strength interference model and they can’t embody the relationship between reliability and the number of load application essentially, namely, they can only calculate the reliability when random load is applied for one or the specified times [1-10]. In this section, we present the reliability models of components when the life measured by the number of load application.

A. Reliability of components with the number of load application as life parameter when strength doesn’t degenerate

When the load, which leading to the failure of static strength, is random and is applied for multiple times, the effect of the number of load application on reliability should be considered. For the failure of static strength, the limited strength is taken as a constant, and in this case it is reasonable in developing the reliability model of components under random repeated load that taking the value of strength as a constant.

When strength doesn’t degenerate or strength degradation is not obvious, it can be regarded as extracting n load samples that random load is applied for n times from the statistic meaning of random load application, and it is reliable for a component or system under these n load samples if it doesn’t fail under the maximum among them. Therefore, the reliability when load is applied for n times equals that corresponding to the maximum among these n load samples applied once, and the maximum can be defined as equivalent load.

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The equivalent load corresponding to these $n$ times load applications, is the maximum order statistic [11], which is determined by the $n$ load samples. And the probability density function of equivalent load is

$$f(x) = nF_n(x)^{n-1}f(x)$$

(1)

Further, the reliability model of components when random load is applied for $n$ times, can be developed as

$$R(n) = \int_0^\infty f(x) [F(x)]^n dx$$

$$= \int_0^\infty f(x) [F(x)]^n \prod_{i=1}^n F(x_i) dx$$

(2)

In Eq. (2), the relationship between reliability and the number of load application is embodied. And it can be applied to calculate the reliability of components when random load acts for the arbitrary times. Specially, for $n=1$, Eq. (2) is same as the conventional load-strength interference model.

**B. Reliability of components with the number of load application as life parameter when strength degenerate**

For fatigue failure, strength degenerates with the number of load application, in other words, the residual strength changes as the number of load application increases. Generally, the rule of strength degradation is dependent on the magnitude of load, the number of load application and the order of load application. When the magnitude of load is constant or its variance is comparatively smaller compared to its mean, the residual strength can be regarded as the function of the magnitude of load and the number of load application only.

Here, we only discuss the case that the variance of magnitude of load is comparatively smaller. And we utilize the residual strength model developed by literatures [12, 13]. When the original strength is $\delta$, the residual strength $\delta_n$ after the $n$ times of load application, can be expressed as

$$\delta_n = \delta - (\delta - s) \left( \frac{n}{N_s} \right)^{\mu}$$

(3)

Here, $s$ is the magnitude of load, $N_s$ is the fatigue life corresponding to the load level $s$, and $\mu$ is the material coefficient.

When the mean of load $\mu$ is constant and variance is smaller, Eq. (4) can be approximately expressed as

$$\delta_n = \delta - (\delta - \mu) \left( \frac{n}{N_{\mu}} \right)^{\mu}$$

(4)

According to Eq. (4), when $\delta$ is deterministic, $\delta_n$ is deterministic, too. Assuming that $A_n$ represents the event that a component doesn’t failure when the $n$th load application is done if it survives from the $n-1$ load applications, the probability that event $A_n$ happens, can be expressed as

$$P(A_n | \delta) = \int_0^{\delta_{n-1}} f(x) dx = F_n(\delta_{n-1})$$

(5)

In the case that $\delta$ is deterministic, the reliability of component after the $n$ times of load application is

$$R(n | \delta) = R(n-1 | \delta) P(A_n | \delta) = \prod_{i=1}^n P(A_i | \delta)$$

$$= \prod_{i=1}^n \int_0^{\delta_{n-1}} f(x) dx = \prod_{i=1}^n F_n(\delta_{n-1})$$

(6)

Further, $\delta_{(-1)}$ in Eq.(6) can be expressed as the function of $\delta$ and $i-1$ by Eq.(4), and Eq. (6) is rewritten as

$$R(n | \delta) = \prod_{i=1}^n F_n(\delta_{i-1})$$

(7)

Generally, original strength $\delta$ is a random variable with the probability density function $f(\delta)$, and then, the reliability of component when random load is applied for $n$ times is

$$R(n) = \int_0^\infty f(\delta) \prod_{i=1}^n F_n(\delta_{i-1}) d\delta$$

(8)

Especially, when strength doesn’t degenerate (namely, $\delta_{n-1}$ is independent of $i-1$), Eq.(8) has the same expression as Eq.(2).

**III. FAILURE RATE MODEL OF COMPONENTS UNDER REPEATED RANDOM LOAD**

When time is as the life parameter of components or systems, the failure rate at time $t$ is defined as the failure probability per unit time of components or systems, which haven’t failed up to time $t$, and the average failure rate during the interval $[t, t+\Delta t]$ can be expressed as

$$h(t) = \frac{R(t) - R(t + \Delta t)}{R(t) \cdot \Delta t}$$

(9)

Further, letting $\Delta t \to 0$ and applying the derivative definition of a limit, the failure rate at time $t$ is derived as

$$h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{R(t) \cdot \Delta t} = \frac{d}{dt} \left( \frac{R(t)}{R(t)} \right)$$

(10)

Similarly, the failure rate model of a component, whose life parameter is the number of load application, is derived as follows.

For the components or systems with the number of load application as life parameter, the failure rate is defined as the failure probability per time of a component or system given that it hasn’t fail up to $n$ times.

According to the above definition, the average interval failure rate $\overline{R(n)}$ during the interval $[n, n + \Delta n]$ can be expressed as

$$\overline{R(n)} = \frac{R(n) - R(n + \Delta n)}{R(n) \Delta n}$$

(11)

Obviously, the minimal number of load application, is $\Delta n_{\text{min}} = 1$, and in general, the total number of load application is large enough that $\Delta n_{\text{min}}$ is a very small compared to it. Then, letting $\Delta n = \Delta n_{\text{min}}$ (namely, $\Delta n = 1$), the failure rate when random load is applied for $n$ times is expressed approximately as

$$h(n) = \overline{R(n)} = \frac{R(n) - R(n + 1)}{R(n)} = 1 - \frac{R(n + 1)}{R(n)}$$

(12)

Further, by substituting Eq. (2) into Eq. (12), the failure rate of components without strength degradation, can be expressed as
Similarly, through substituting Eq. (8) into Eq. (12), the failure rate of components with strength degradation, can be expressed as

\[
h(n) = \frac{\int_0^{+\infty} f_\delta(\delta)[1-F_i(\delta,n)] \prod_{i=1}^{n} F_i(\delta,i-1) \ d\delta}{\int_0^{+\infty} f_\delta(\delta) \prod_{i=1}^{n} F_i(\delta,i-1) \ d\delta}
\]  

(13)

IV. STUDY ON FAILURE RATE OF COMPONENTS UNDER REPEATED RANDOM LOAD

In the following, we study the relationship between the failure rate of components and the number of load application in different cases. Taking a component with single failure mode as an example, strength subjects to the normal distribution with mean 600MPa and standard deviation 60MPa, and load follows the normal distribution with mean 400MPa and standard deviation 40MPa. When strength doesn’t degenerate as the number of load application, the curves for reliability and failure rate are shown as Fig.1 and Fig.2, respectively.

![Figure 1. Relationship between reliability of components and the number of load application (without strength degradation)](image1)

![Figure 2. Failure rate curve of components (without strength degradation)](image2)

When strength degenerates as the number of load application, and strength degradation follows the linear rule as

\[
\delta_{(n)} = \delta - \left(\delta - \mu \right) \left(\frac{n}{150000}\right)^{1.5}
\]

the curves for reliability and failure rate of components are shown as Fig.3 and Fig.4, respectively.

![Figure 3. Relationship between reliability of components and the number of load application (with strength degradation)](image3)

![Figure 4. Failure rate curve of components (with strength degradation)](image4)

From the Fig. 1 and Fig. 2, it can be concluded that when strength doesn’t degenerates, reliability still decreases as the number of load application, and the failure rate decreases, too. The failure rate has the partial feature of bathtub-shaped curve. From Figs. 3 and 4, it can be concluded that when strength degenerates, reliability decreases more obviously as the number of load application, and the failure rate has the whole feature of bathtub-shaped curve, with initial failure period, random failure period and wear-out period.

![Figure 5. Curves of failure rate under different dispersion of random load (without strength degradation)](image5)
From Fig. 5, even though strength doesn’t degenerate, the failure rate of components decreases as the number of random application increases. As the conventional explanation, the early higher failure rate is caused by the flaws related with strength only. In fact, it can be seen that when the mean of load is fixed, the larger the deviation is, the higher the failure rate is. Therefore, the randomness of load is of important effect on the failure rate as strength, and it is also one of main factors that cause the early higher failure rate [1, 14].

V. CONCLUSIONS

In this paper, we developed the failure rate model of components with the number of load application as the life parameter based on the load-strength theory, and study the relationship between reliability of components and the number of load application, and that between the failure rate and the number of random load application in different cases (without and with strength degradation), respectively. The result shows that strength doesn’t degenerate, both reliability and failure rate still decrease as the number of load application, and when strength degenerates, reliability decreases more obviously and the failure rate presents the bathtub-shaped curve as the number of random load application increases. The failure rate under the cases of different dispersion load is studied, and it is seen that the randomness of load is of the same important effect on the failure rate as strength, and it is one of main factors resulting in the early higher failure rate.

The models developed can embody the effect of the number of random load application on reliability and failure rate of components, and can be used to direct the reliability-based design, reliability test, maintenance, et al.

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