A Study on Comprehensive Evaluation Model of Product Producing Process Capability

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Abstract—Process Capability Index is a widely used tool for quality evaluation. Available methods of Process Capability evaluation are limited to machining process. There are no suitable methods for comprehensive evaluation of total producing process capability. In this paper, a comprehensive evaluation model of product producing process capability is proposed, so that data of all stages can be fully used. Methods for machining capability evaluation are extended to purchasing, assembling and delivery inspection and an index system of producing process capability evaluation is presented. In addition, analytic hierarchy process is used to determine the weight of each stage and then producing process capability index is achieved by summing. Finally, an application example is analyzed to show that the entire producing capability can be evaluated objectively. It should play an important part in monitoring and improving producing process capability.

Keywords—Producing Process; Quality Evaluation; Process Capability Index; Comprehensive Evaluation Model

I. INTRODUCTION
Quality of products derives from design, forms in producing and is shown in using. Producing is an important process for quality of products in life cycle. Producing process includes purchasing, machining, assembling and delivery inspection [1]. Evaluation of products quality has changed from sampling inspection to evaluation of products process capability. Process Capability refers to actual machining accuracy of process when it is stable in a period of time. It can reflect capability of process in machining. Process Capability Index is a convenient tool for quality evaluation. It can evaluate inner quality of products quantitatively in time and reflect production craft level and yield of normal condition. Therefore, it is widely used in various fields.

Engineers and scientists have fully studied the capability of multiple characteristics and processes of machining. For instance, a model of multi-characteristics process capability based on weight [2], evaluation of multi-characteristics process [3], calculation method for multivariate process capability indices [4], evaluation of process capability for a machining center [5], process capability evaluation for the process of product families [6] and so on. However, existing studies are only for machining process and there are no evaluation models for total producing process. In this paper, methods of machining capability evaluation are extended to purchasing, assembling and delivery inspection by using system engineering idea. In addition, analytic hierarchy process is used to determine the weight of each stage. What’s more, a comprehensive evaluation model of products producing process is proposed. Therefore, it can be reference for evaluation and monitoring of producing process comprehensive quality assurance.

II. COMPREHENSIVE EVALUATION MODEL OF PRODUCT PRODUCING PROCESS CAPABILITY

A. Evaluation Indices System
Producing process includes four stages, purchasing, machining, assembling and delivery inspection. Each stage has many units of product, and each unit has many processes and characteristics. The overall idea for evaluation of products producing process capability is showed as follows. Critical units and their critical processes and characteristics should be determined in prepare procedure. Firstly, evaluation method for multivariate process capability indices is used for critical process to get its capability index. Secondly, geometrical average method is adapted to get critical multi-processes capability index. Thirdly, capability indices of critical units are evaluated comprehensively to get capability index of each stage. Finally, product producing process capability index is got by comprehensive evaluation. Note that steps in different stages may be different.

Evaluation indices system for product producing process capability is showed in Fig. 1.
Based on the evaluation indices system, after determining the critical units and their critical processes and characteristics of each stage, detailed steps for calculation of process capability index of each stage is as follows.

B. Evaluation for Capability of Critical Unit of Each Stage

1) Evaluation for Capability of Homemade Parts in Machining

There are two steps in evaluation of capability of critical homemade parts.

First, using evaluation method for multivariate process capability indices for critical process of critical unit to get its capability index.

In this paper, evaluation method for multivariate process capability indices proposed by Shao Xi Wang and Xin Zhang Jia [3] is adapted. It is described as follow.

Define $X$ as $v \times n$ sample matrix, where $v$ is number of characteristics, which have specification limits, and $n$ is sample size. Suppose that data of characteristics obey the normal distribution. Then statistic $(X - \mu_0)^T \Sigma^{-1}(X - \mu_0)$ obeys $\chi^2$ distribution. Define the region formed by $(X - \mu_0)^T \Sigma^{-1}(X - \mu_0) \leq \chi^2_{v, \alpha}$ as process region, where $m$ denotes dimension and $\Sigma$ denotes covariance matrix. Suppose that the data obey multivariable normal distribution, whose probability isoline is ellipse. Define multi-processes capability index as

$$C_{pm} = \left[\frac{Vol(R_1)}{Vol(R_2)}\right]^{\frac{1}{2}}$$

In (1), $R_1$ is area or volume of specification region and $R_2$ is area or volume of modified process region. Modified process region is defined as region that has the same shape as specification region and has inscribed ellipse isoline, which has certain probability.

For two-dimensional normal distribution, modified process region is a rectangle, whose boundary defines the upper and the lower specification limit, $UPL_i$ and $LPL_i$, $i=1,2,\ldots,v$ respectively.

$$UPL_i = \mu_i + \sqrt{\chi^2_{v, \alpha} \det(\Sigma_i)} \left\{ \begin{array}{l} \det(\Sigma^{-1}) \\ \det(\Sigma_i) \end{array} \right.$$

$$LPL_i = \mu_i - \sqrt{\chi^2_{v, \alpha} \det(\Sigma_i)} \left\{ \begin{array}{l} \det(\Sigma_i) \\ \det(\Sigma^{-1}) \end{array} \right.$$
Second, using geometrical average method for critical multiple processes of critical unit to get critical multi-processes capability index.

Suppose that processes are independent. Using geometrical average method proposed by K.T.Yu[5] to define multi-processes capability index \( C_{pm}^{T1} \). It is defined as in (5).

\[
C_{pm}^{T1} = \prod_{i=1}^{q} \left( \prod_{l=1}^{n} \left( \frac{USL_l - LSL_l}{UPL_l - LPL_l} \right)^{w_i} \right)^{1/n} \tag{5}
\]

In (5), \( w_i \) denotes the weight of the i process, whose range is 1,2,3,4,5.

According to K. S. CHEN[6], when characteristics obey normal distribution, the relationship between yield and process capability can be described as in follows.

\[
C_{pm} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} \tag{6}
\]

If \( C_{pm} = c \), then (7) founds.

\[
p = \frac{\Phi \left( \frac{1+\sqrt{\frac{1}{\sigma^2}(\sigma/d)^2}}{\sigma/d} \right) - \Phi \left( \frac{1-\sqrt{\frac{1}{\sigma^2}(\sigma/d)^2}}{\sigma/d} \right)}{1} \tag{7}
\]

In (6) and (7), \( \sigma \) is process standard deviation, \( d = \frac{USL - LSL}{2} \), \( \mu \) is process mean, \( T \) is process target, and USL and LSL are the upper and the lower specification limit respectively.

2) Evaluation for Capability of Critical Units in Other Stages

To guarantee high inner quality and reliability of purchased parts and materials, the company can demand supplier provide data about process capability index. Data of purchasing stage include quality characteristics data of purchased parts and materials and yield. If quality characteristics data are known, evaluation method for multivariate process capability indices proposed by Shao Xi Wang and Xin Zhang Jia[3] is adapted to get the capability index, and the step is the same as the first step in 1). If there is only yield for some characteristic, when it obeys normal distribution and process mean is equal to process target, the relationship between yield and process capability is that

\[
C_{pm} = \frac{1}{3} \Phi^{-1} \left( \frac{1+p}{2} \right) \tag{8}
\]

In (8), \( p \) is yield. Therefore, process capability index can be obtained.

Assembling is composed of many processes, so the evaluation is similar to evaluation of machining. The difference is that using evaluation method for multivariate process capability indices for parts to be assembling to get the capability index in the first step.

Delivery inspection refers to testing, experimental verification of performance indices and quality inspection of delivery. It includes four degrades, A, B, C, D. Grade A is common inspection, and it verifies main performances of products for some batch or the whole. Grade B is particular sampling inspection, and it refers to verification of specific performance under special environment. Grade C is environmental suitability inspection, and it refers to experimental verification in simulating work condition. Grade D is reliability experimental verification, and it refers to capability of product finishing specific performances under specific conditions. Note that grade C and D are not necessary for plot produce. Capability index of delivery inspection stage can be obtained as follows. Firstly, use evaluation method for multivariate capability indices to get capability index of grade A and the step is the same as the first step in 1). Then, inspection of grade A, B, C, D can be treated as four processes. Therefore, multi-processes evaluation can be adapted to get capability index of delivery inspection, and the step is the same as the second step in 1).

C. Comprehensive Evaluation

Based on capability indices of critical units in each stage, capability index of each stage can be obtained. Then, integrate index of each stage to get the capability index of total process for the product. The detailed steps are described as follows.

First, integrate indices of critical units of each stage to get capability index of each stage.

\[
C_{pm}^{T2} = \sum_{i=1}^{k} w_i C_{pm}^{Tui} \tag{9}
\]

In (9), \( C_{pm}^{Tui} \) denotes multi-processes capability index of the i unit (i=1…..k), and \( w_i \) denotes the weight of the i product, and its range is [0, 1].

In this paper, analytic hierarchy process[7] is used to determine the weight of critical units of each stage. Using this method, subjective experience and objective evidence are combined to obtain scientific and rational result. Suppose that there are n critical units in a stage. Firstly, any two units are compared to form judgment matrix. Then, the maximum eigenvalue and its eigenvector are solved. In addition, consistency check should be done. If the judgment matrix
passes the consistency check, the eigenvector should be normalized. The normalized eigenvector is vector of weights of \( n \) critical units. The detailed steps will be described at section 3. Table 1 displays the scales and their meanings of making judgment matrix.

### Table I. Scales and Meanings of Judgment Matrix

<table>
<thead>
<tr>
<th>Scales</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Two units have the same importance</td>
</tr>
<tr>
<td>3</td>
<td>The former is a little bit important than the latter</td>
</tr>
<tr>
<td>5</td>
<td>The former is obvious important than the latter</td>
</tr>
<tr>
<td>7</td>
<td>The former is strong important than the latter</td>
</tr>
<tr>
<td>9</td>
<td>The former is extreme important than the latter</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Middle values of adjacent values above</td>
</tr>
</tbody>
</table>

For judgment matrix \( C \),

\[
C_{ij} = \frac{1}{C_{ji}} \quad (i \neq j), \quad C_{ii} = 1(i = 1...n) \quad (10)
\]

To check consistency of judgment matrix, consistency index (CI) should be calculated as in (11).

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1} \quad (11)
\]

The smaller of CI, the better consistency of the judgment matrix. Because of different exponents of matrixes, the average random consistency index (RI) is needed. For judgment matrixes, which have exponents of 1-9, values of RI are showed in table 2.

### Table II. Average Random Consistency Index

<table>
<thead>
<tr>
<th>Exponent</th>
<th>RI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.58</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
</tr>
<tr>
<td>5</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
</tr>
<tr>
<td>7</td>
<td>1.32</td>
</tr>
<tr>
<td>8</td>
<td>1.41</td>
</tr>
<tr>
<td>9</td>
<td>1.45</td>
</tr>
</tbody>
</table>

When random consistency ratio \( CR = \frac{CI}{RI} < 0.1 \), the judgment matrix is considered having satisfying consistency. Otherwise, the judgment matrix should be adjusted to have satisfying consistency. Note that judgment matrixes of one and two exponents always have complete consistency.

Second, integrate capability index of each stage to get producing process capability index.

In this paper, method proposed by K. S. CHEN[6] is used to define producing process capability index as in (12).

\[
C_{pm}^{CT} = \min \{C_{pm}^{T1}, C_{pm}^{T2}, C_{pm}^{T3}, C_{pm}^{T4}\} \quad (12)
\]

In (12), \( C_{pm}^{Tj} \) (\( i = 1,2,3,4 \)) denotes capability index of the \( i \) stage. Yield is defined as in (13).

\[
p^{CT} = \sum_{i=1}^{4} w_i p_{i}^{T2} \quad (13)
\]

In (13), \( p_{i}^{T2} \) denotes yield of the \( i \) stage, and \( w_i \) denotes weight of the \( i \) stage, and its range is \((0,1)\). In this way, we can guarantee that yield \( p^{CT} \geq 2\Phi(3C) - 1 \), when \( C_{pm}^{CT} = C > 1/3 \).

### III. Case Study

#### A. Calculation

Take the producing process of some product for example. Table 3 displays critical units and their critical characteristics and processes of each stage, where A1 is purchased part, A2 is material; B1, B2, B3 are all homemade parts; C1 and C2 are assemblies, C11 and C21 are sub-assemblies, C111, C211 and C212 are critical assembling characteristics of parts; D1 is the product, D11 and D12 are common inspection and particular sampling inspection respectively, D111, D112 and D121 are critical checking characteristics.

### Table III. Producing Process of Some Product

<table>
<thead>
<tr>
<th>Stage</th>
<th>Purchasing</th>
<th>Machining</th>
<th>Assembling</th>
<th>Delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1 A2</td>
<td>B1 B2 B3</td>
<td>C1 C2</td>
<td>D1 D11</td>
</tr>
<tr>
<td></td>
<td>- -</td>
<td>B11 B12</td>
<td>B21 B31 B32 B33 C1 C2</td>
<td>D11 D12</td>
</tr>
<tr>
<td></td>
<td>A11 A111</td>
<td>B111 B112 B121 B122 B21 B211 B212 B31 B311 B32 B321 B33</td>
<td>C1 C11 C2 C21</td>
<td>D11 D112 D121</td>
</tr>
</tbody>
</table>

Use the model proposed in section 2 to evaluate the producing process capability of the product. The steps are as follows.

First, obtain capability index of critical process for critical unit of each stage by calculation.

Take the calculation of the capability index of A1 for example. The data is showed in table 4.

### Table IV. Characteristic Data of A1

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>99</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11</td>
<td>2.95</td>
<td>2.98</td>
<td>...</td>
<td>3.02</td>
<td>2.97</td>
</tr>
<tr>
<td>A112</td>
<td>3.01</td>
<td>2.97</td>
<td>...</td>
<td>3.03</td>
<td>3.01</td>
</tr>
</tbody>
</table>

The upper and the lower specification limit of A111 are 3.5 and 2.5 respectively. The upper and the lower specification limit of A112 are 3.55 and 2.45 respectively. We can get the following results by calculation. The sample means of A111 and A112 are 2.98 and 3.005 respectively, and sample standard deviation are 0.1628 and 0.2102 respectively. The covariance matrix of the sample is \( \Sigma = \begin{pmatrix} 0.0265 & 0.0292 \\ 0.0292 & 0.0442 \end{pmatrix} \), and its inverse matrix is \( \Sigma^{-1} = \begin{pmatrix} 138.7058 & -91.6337 \\ -91.6337 & 83.1607 \end{pmatrix} \), \( \kappa_{2,(0.975)}^2 = 11.829 \). Then area
of specification region can be obtained, \( R_{T0} = (3.5-2.5) \times (3.55-2.45) = 1.1 \). The area of modified process region is as in (14).

\[
R_{PM} = 2\sqrt{11.829 \times 83.1607 \times 3138.1} = 1.6194 \quad \text{(14)}
\]

The process capability index is as in (15).

\[
C_{pm} = \left( \frac{R_{T0}}{R_{PM}} \right)^{1/2} = \sqrt{\frac{1.1}{1.6194}} = 0.8242 \quad \text{(15)}
\]

In a similar way, capability indices of other processes can be got. They are showed in table 5.

<table>
<thead>
<tr>
<th>TABLE V. CAPABILITY INDEX OF PROCESSES OF CRITICAL UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process Unit</td>
</tr>
<tr>
<td>Capability Index</td>
</tr>
</tbody>
</table>

Second, obtain capability index of critical unit of each stage by calculation.

Weights of processes of critical unit are determined by experts, as is shown in table 6.

<table>
<thead>
<tr>
<th>TABLE VI. WEIGHTS OF PROCESSES OF CRITICAL UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
</tr>
<tr>
<td>Weight</td>
</tr>
<tr>
<td>Process</td>
</tr>
<tr>
<td>Weight</td>
</tr>
</tbody>
</table>

Compute multi-processes capability indices for B1, B3 and D1 as in (16), (17) and (18).

\[
C_{pm}^{B1} = (1.0763 \times 0.8685)^{1/2} = 1.0201 \quad \text{(16)}
\]

\[
C_{pm}^{B3} = (1.0956 \times 1.0002 \times 1.0828)^{1/2} = 1.0383 \quad \text{(17)}
\]

\[
C_{pm}^{D1} = (1.1983 \times 1.2657 \times 1.2657)^{1/2} = 1.2485 \quad \text{(18)}
\]

Then capability indices of all critical units are obtained. They are showed in table 7.

<table>
<thead>
<tr>
<th>TABLE VII. CAPABILITY INDEX OF CRITICAL UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
</tr>
<tr>
<td>Capability Index</td>
</tr>
<tr>
<td>Unit</td>
</tr>
<tr>
<td>Capability Index</td>
</tr>
</tbody>
</table>

Third, obtain capability index of each stage by calculation.

The judgment matrix of each stage is as in (19).

\[
A = \begin{bmatrix} 1/2 & 1/2 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1/5 & 1/5 & 1/3 \\ 3 & 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 \\ 1/3 & 1 \end{bmatrix} \quad D = [1] \quad \text{(19)}
\]

Compute their maximum eigenvalues and their corresponding eigenvectors. Then normalize the eigenvectors. The results are as in (20), (21), (22) and (23).

\[
\lambda_{max} = 2, \quad W_p = (0.110, 0.581, 0.309) \quad \text{(20)}
\]

\[
CI = 3.0037, \quad RI = 0.58, \quad CR = \frac{CI}{RI} = 0.003 < 0.1 \quad \text{(22)}
\]

\[
\lambda_{max} = 2, \quad W_c = (0.75, 0.25) \quad \text{(23)}
\]

Take machining stage for example. Its process capability index is as in (24).

\[
C_{pm} = 0.9600 \quad \text{(24)}
\]

In a similar way, capability indices of other stages can be obtained. Additionally, yield of critical unit can be got based on the relationship between process capability index and yield. They are showed in table 8.

<table>
<thead>
<tr>
<th>TABLE VIII. CAPABILITY INDEX AND YIELD OF EACH STAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage</td>
</tr>
<tr>
<td>Capability Index</td>
</tr>
<tr>
<td>Yield</td>
</tr>
</tbody>
</table>

Fourth, obtain comprehensive capability index of product producing process by calculation.

The judgment matrix of the product is as in (25).

\[
G = \begin{bmatrix} 1 & 1/5 & 1/3 & 1/2 \\ 5 & 1 & 2 & 3 \\ 3 & 1/2 & 1 & 2 \\ 2 & 1/3 & 1/2 & 1 \end{bmatrix} \quad \text{(25)}
\]
Compute its maximal eigenvalue and corresponding eigenvector. Then normalize the eigenvector. The results are as follows.

\[ \lambda_{\text{max}} = 4.0145, \ W = (0.088, 0.483, 0.272, 0.157) \]  

Therefore, the producing capability index of the product can be obtained as in (28).

\[ C_{\text{pm}}^{CT} = \min\{0.9600, 0.9842, 1.0097, 1.2485\} = 0.9600 \]  

The yield is that

\[ p^{CT} = 0.088 \times 0.995483 + 0.483 \times 0.996712 + 0.272 \times 0.997346 + 0.157 \times 0.999755 = 0.997254 \]  

B. Analysis

It can be seen from the case that yield is only 99.7254%, namely, 3σ level isn’t reached, although capability index of delivery inspection is high. The reason is that capability index of A1 is too low, so the capability index of purchasing stage is low. Besides, capability indices of homemade part B2 of machining stage and assembly C1 of assembling stage are also not high. Consequently, the total process capability index is a little low. The evaluation result is suggested that reinforce the management of A1 or change its supplier, improve process B21 and assembly C11.

IV. CONCLUSION

Quality is the focus of the manufacturing company, and process capability index is a widely used tool for quality evaluation. To use inspection data of all stages of producing to evaluate the quality objectively, a comprehensive evaluation model of product producing process capability is proposed in this paper. The model is proved to be capable of evaluating capability of total producing process objectively by an unabridged case. In addition, it plays an important role in finding and solving quality problems and improving the quality level.

It is supposed that characteristic obeys normal distribution in the calculation of the model, but there are many non-normal distributions in practice. Therefore, the use of the model is limited some how. So the evaluation methods of producing process capability index under non-normal distribution is still worth being studied further.

REFERENCES