Rational Design of Conditions for Reliability Test

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Abstract—This article deals with the problems associated to practical determining the reliability test scope. The author presents his own findings and experience resulting from his participation in testing reliability of many technical systems. Main attention is dedicated to procedures for ascertaining reasonable limits to be considered for performing reliability tests. The author presents main parameters of reliability tests, which predetermine of the course of a test. Among other things, the following test parameters are discussed: value of measure tested, time of test, confidence level, and number of tested product, etc.

Keywords-reliability test; confidence level; time of test; reliability measure; reliability test conditions;

I. INTRODUCTION

In assuring that product dependability is achieved, the reliability tests have a primary role as they provide to verify whether the product fulfills the reliability requirements at the early life stages or, what is the achieved level of the reliability of product. Recently, in the environment of the Czech Republic industry we more often see that clearly specified requirements for reliability are integrated into the contracts. Consequently, it is then requested that fulfillment of contractual requirements for reliability are verified through the appropriate tests. In general, it is also required that such verification is performed with a certain pre-defined confidence.

Currently, to fulfill the above-mentioned task, i.e. to verify accomplishing or non-accomplishing to meet a certain measure at a pre-assigned level of confidence does not imply any principal problem. There are many procedures and methods available, most of them standardized, that provide conducting the reliability tests under clearly defined conditions and to assess them objectively – refer [1] or [2].

However, in practice we can rather often see the cases when the scope (conditions) of the test (test time, amount of tested products, desired level of confidence, etc.) is established irrespective of the character of tested parameter, which aggravates or disallows to really verify whether the product meets the established requirements. There are even the cases of serious disputes between the contractor and purchaser as to whether the product in question meets or not the reliability requirements. The purpose of this article is to point out these problems and to indicate how they could be addressed. This article is based on the author’s practical experience with the reliability tests on a series of technical systems.

The extent of this article is limited and therefore we will focus on cases when the random variable considered (time to failure, mean time between failures, etc.) has an exponential distribution. The below-mentioned conclusions and recommendations can be appropriately applied also for other types of distribution.

II. SCOPE OF THE RELIABILITY TEST

The reliability tests to be properly interpreted, the term „test scope” is to be used. The test scope is understood to be a set of parameters, which describe the test in relation to the confidence level of test assessment. In general, test scope can be characterized by a set of the following data:

- time of test duration in a real time („watch is used“),
- accumulated test time,
- number of products tested in a set (in a sample),
- amount of failures occurred in a set of n products during a test,
- desired confidence level of the test assessment.

The test scope, i.e. actual values of individual values must be established with respect to the required values of reliability measures that are to be verified by the test. Interrelations of individual parameters characteristic for the test scope are based on calculations for interval estimates of reliability measures based on an assumption of a constant failure rate (exponential distribution). Further, only calculations for one-sided interval assessment will be mentioned, which are most frequently used in practice [3]. For the upper confidence limit of failure rate shall apply:

\[
\lambda_U \leq \frac{\chi^2_p (\nu)}{2T}
\]  

(1)

And for lower confidence limit of mean operating time between failures shall apply:

\[
MTBF_L \geq \frac{2T}{\chi^2_p (\nu)}
\]  

(2)
where $T$ is the accumulated test time, $\lambda_U$ is the upper confidence limit of failure rate, $MTBF_U$ is the lower confidence limit of mean operating time between failures, $\chi^2_p (\nu)$ is the $p$ fractile of the $\chi^2$ distribution with $\nu$ degrees of freedom, $p$ is the confidence level of reliability assessment, and $\nu$ is the number of degrees of freedom.

Number of degrees of freedom depends on number of failure during a test, type of confidence limit, and roles of tests. For example for time truncated tests in the case when a products are replaced after a failure number of degrees of freedom for lower confidence limit of $MTBF$ can be determined using the following equations [1]:

$$\nu = 2r + 2$$

where $r$ is the number of failures during the test.

Note that an accumulated test time $T$ in the relation (1) or (2) does not represent a time of test duration measured using a „watch“, but it represents a total cumulative test time (sum of the operating times of all products tested), which is dependent on the number of tested products and the test plan used. The accumulated operation time can be determined by the following equation:

$$T = \sum_{i=1}^{i=n} t_i$$

where $t_i$ is the real operation time of $i$-th product in test, and $n$ is the number of tested products.

### III. Test Plan and Equivalent Test Time

The course of testing is characterized by the so called test plan, which is a set of rules to codify a method how the test is performed. It indicates an extent of test sample, method of replacement or repair of a product failed during the test and a method of termination of the whole test. It is symbolically recorded as arranged triples of symbols $[n, (U$ or $R$ or $M), (r_0$ or $\tau_0$)]. The first symbol $n$ indicates a number of products used on the test. The second symbol indicates activities upon failure:

- $U$ - upon failure, the product is removed from the test and it is not replaced with any other product,
- $R$ - upon failure, a product is replaced with a new one,
- $M$ - upon failure, a product is repaired and returned into test.

The last symbol represents a way of termination of the test:

- $r_0$ - indicates a number of failures per test. The test will terminate when $r_0$-th failure occurs,
- $\tau_0$ - indicates the test time. The test will end when a time $\tau_0$ is achieved.

Every test is conducted with a limited number of products $n$. The test will terminate either after a failure of all products tested, or after a certain time of test duration $\tau_0$ or, after an occurrence of a certain number of failures $r_0$. In principle, the test plans can be divided according to the nature of the set of data collected from the test into four basic groups:

- The test results produce a complete set – this group includes cases when during the test, all tested products fail and products are neither replaced nor corrected upon failure. Officially, these plans are known as $[n, U, n]$.
- The test results produce a failure truncated set – the so called $r$ - plans. The test will encompass $n$ of same products. The test will terminate upon a pre-determined number of failures $r_0$. The faulty components are either replaced with new ones during the test $[n, R, r_0]$ or they are not replaced $[n, U, r_0]$, or they are repaired $[n, M, r_0]$. In this case, the random variable is a time of test $\tau$.
- The test results produce a time censored set - the so called $t$ - plans. The test will encompass $n$ of same products. The test will terminate after a pre-determined time of testing $\tau_0$ elapses. The faulty components are either replaced with new ones during the test $[n, R, \tau_0]$ or they are not replaced $[n, U, \tau_0]$ or they are repaired upon failure $[n, M, \tau_0]$. In this case, the random variable is a number of failures $r$ occurring during the test.
- The test results produce a progressively truncated set. These are mixed plans of a type $[n, R, r_0]$ and $[n, M, \tau_0]$ truncated randomly by a number of failures or the time of the test. These tests according to these plans provide the sets of data of various lengths and of various types of termination. One sub-group of all observed intervals is terminated after a failure, the second sub-group by the time of observation (without a failure).

Objective analysis of the given types of sets of measured data is a key to properly select a procedure how to determine equivalent times of the test. To this objective, we must distinguish two cases:

- tests during which no failures occur (failure-free test),
- tests when failures occur.

Nevertheless, the equivalent time of test always represents a pure sum of the times of operation of all products placed on test. General relations indicating a relation between the accumulated time of the test and the test time (measured using the watch) for individual types of test plans are mentioned in references [3].

For example, when testing non-repairable products according to the test plan $[n, U, r_0]$ (products are not replaced upon failure, the test will be wound up when $r_0$-th failure occurs) the equivalent time of test is calculated using the following equation:
where \( T \) is the test time, i.e. time from the beginning of the test up to the occurrence of \( r_0 \)-th failure, \( t_i \) is the time to failure of \( i \)-th product, and \( r_0 \) is the determined number of failures upon which the test terminates.

\[
T = \sum_{i=1}^{n} t_i + (n-r_0) \tau 
\]  

where \( \tau \) is the test time, i.e. time from the beginning of the test up to the occurrence of \( r_0 \)-th failure, \( t_i \) is the time to failure of \( i \)-th product, and \( r_0 \) is the determined number of failures upon which the test terminates.

IV. DETERMINATION OF THE TEST SCOPE WITHOUT FAILURES

Establishment of the scope of the reliability tests rests in determining the parameters that are characteristic for the course of a test and a method of its assessment. To determine these parameters, it is necessary to use relations of equation (1) or (2) and to respect a character of studied values. This article also discusses typical problems related to establishing the test scope and it indicates possibilities how to solve them.

Use of equation (1) or (2) enables to determine a certain minimum equivalent time of test during every test, during which accomplishment of appropriate reliability requirements at pre-assigned level of confidence can be proved. A prerequisite to accomplish the test task during this minimum equivalent time of test is that during the test no failure will occur.

This potential possibility to minimize a time necessary to perform a test in practice often requires conducting a test just in this way. The reason is to reduce costs of testing and to shorten delivery dates. The accumulated time of test needed to prove that at the defined level of confidence, a required value of reliability measure was achieved, provided a test was free of failures, can be calculated from the following equation:

\[
T_{REL} = \frac{T}{m_D} = \frac{\chi^2_r(2)}{2}
\]  

where \( T_{REL} \) is the relative length of accumulate test time (related to a desired value of reliability measure), and \( m_D \) is the desired value of reliability measure (MTBF, MTTF).

Thus, a relative length of equivalent time of the test will depend on the level of desired confidence. Tab. 1 shows how the length of equivalent time of the test changes with the value of confidence. The last row of this table gives the expected probability that the test will terminate without a failure under the assumption that a real value of reliability measure will equal to the desired value \( (m=m_D) \).

The following example demonstrates how important the values given in Table 1 are. If we wish to prove with a 95% confidence that the desired value of reliability measure have been achieved, then product must work time equaling to a triple of a value \( m_D \) without failure during the test. The probability that the test will terminate with such a result equals to 5%. In other words, there is 95% probability that failure can be expected during the test.

From the above-mentioned text it is evident that the assumption of a failure-free run of the test brings a relatively high probability of an unsatisfactory test even if a product meets the established reliability requirements.

The assumption of the failure-free test can be regarded as justifiable only in cases when lower values of confidence are established and thus it can be reasonably expected that the product has a substantially higher level of reliability than that one to be proved by the test.

V. ESTABLISHMENT OF THE SCOPE OF TEST WITH FAILURES

For tests where an occurrence of failures is expected we start from the appropriate test plan in establishing the test scope. In case of \( r \)-plan, the test time is a random variable, because the test is wound up upon an occurrence of a certain pre-defined amount of failures \( r_0 \). To estimate a true time of the test will then depend on the fact whether the products are replaced (repaired) or not upon failure. If the products are replaced upon failure, the expected time of the test can be calculated from the following relationship [4]:

\[
E(t) = \frac{r_0 m}{n}
\]  

where \( E(t) \) is the expected time of the test (measured by watch), \( m \) is the verified reliability measure \( (\text{MTBF, MTTF}) \), \( n \) is the number of products placed on test, and \( r_0 \) is the permissible amount of failures. If the products are not replaced upon failure, the expected test time can be established from the relationship [4]:

\[
E(t) = m \sum_{i=0}^{\infty} \frac{1}{n-i+1}
\]  

In case of \( t \)-plans, determining the time of the test is not a problem as it is pre-determined in the test plan and after this time elapses, the test terminates. In proposing the scope of the test, a general rule will apply – the larger is the set of data (on failures) available, the more credible are the results of statistical evaluation. Consequently, the longer is the reliability test, the more credible are the results. The graphical representation in Fig. 1 shows a dependence of the proved value of reliability measure \( m \) on an equivalent time of the test and the desired level of confidence. The scope of the test should be always established so that it corresponds both to the
value of reliability measure, the attainment of which is to be proved, and to the desired level of confidence.

The level of confidence of the assessment of reliability test should be selected appropriate to purpose of the test and to its scope. The use of unreasonably high levels of confidence \( p \to 1 \) will usually cause that the test fails even in cases when the tested reliability measure meets the established value.

VI. CONCLUSION

In establishing conditions of the reliability test, the mutual relations between individual parameters of the test should be taken into account and the scope of the test should be so as to be really executable. At the same time, it should be also considered, whether with respect to the assumed level of reliability of the product tested there is an acceptable probability that during the test, the accomplishment of requirements would be proved (if they are really accomplished).

At the end, it should be noted that even at the expense of the extreme enhancement of the scope of the test, it is not possible to prove what really does not exist, i.e. the reliability level which is not intrinsic to the product.

ACKNOWLEDGMENT

The work presented in this paper is sponsored by the Ministry of Defence of the Czech Republic (Institutional Research Plan No. M00FVT0000401).

REFERENCES


