Success Ratio Sequential Test Plan Using Development Test Data

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Abstract—Success ratio sequential sampling plan is a common test plan for reliability compliance test. The standard sampling inspection used in practical project for reliability compliance test is difficult for application due to the cost and total sample size, especially for complex systems. While the test data acquired in product development phase is abundant. Thus using these data in reliability compliance test can reduce sample size and cost. This paper adopted a method called success ratio sequential sampling plan, which transformed the data acquired in product development phase to the equivalent actual data using "similarity coefficient", and integrated the equivalent actual data to the real compliance test data using Bayesian statistical formula. A simulation revealed that the data acquired in product development phase could be effectively used and the sequential compliance test sample size could be reduced compared with the traditional compliance test.

Keywords—Reliability Compliance Test, Sequential Test, Success Ratio, Development Test Data

I. INTRODUCTION

Reliability compliance test plays a significant role in reliability engineering. Currently, reliability compliance test plan for binomial products is established mostly based on GB5080.5-85. The test plan established in this way usually needs plenty of samples and thus test cost is very high. At the same time many development tests have been carried out in development phase. Massive test data is acquired. Thus adequately using test data in development phase for establishing test plan of reliability compliance test can reduce test samples and test cost. Aiming at this circumstance, a success ratio integrated sequential sampling plan was proposed, which established reliability compliance test plan based on the trait of success ratio sequential test using development test data in products’ development phase, and reduced the amount of necessary test samples.

II. SUCCESS RATIO SEQUENTIAL SAMPLING PLAN

"Sequential sampling" is a sampling plan, which samples individual one by one and doesn’t fix the numbers until an acceptance or rejection decision can be made based on the rule set beforehand.

Assume that \( p_0 \) is producer’s risk quality, \( p_1 \) is consumer’s risk quality, \( \alpha \) is producer’s risk, \( \beta \) is consumer’s risk, \( n \) is the number of products for compliance test, and \( r \) is the number of failure. Based on Wald approximate formula the success ratio sequential sampling plan can be described by

\[
\begin{align*}
\text{when} & \quad \frac{p_0^r (1 - p_1)^{n-r}}{p_0^r (1 - p_1)^{n-r}} \geq \frac{1 - \beta}{\alpha}, & \text{reject} \\
\text{when} & \quad \frac{\beta}{1 - \alpha} \frac{p_0^r (1 - p_1)^{n-r}}{p_0^r (1 - p_1)^{n-r}} < \frac{1 - \beta}{\alpha}, & \text{continue test} \\
\text{when} & \quad \frac{p_0^r (1 - p_1)^{n-r}}{p_0^r (1 - p_1)^{n-r}} \leq \frac{\beta}{1 - \alpha}, & \text{accept}
\end{align*}
\]

Logarithm is taken to the above formula, which can be expressed by

\[
\begin{align*}
\text{when} & \quad r \geq h_0 + s, & \text{reject} \\
\text{when} & \quad -h_0 + s < r < h_0 + s, & \text{continue test} \\
\text{when} & \quad r \leq -h_0 + s, & \text{accept}
\end{align*}
\]

Where

\[
\begin{align*}
h_0 &= \frac{\ln \frac{1 - \alpha}{\beta}}{\ln \frac{p_0}{p_1} + \ln \frac{1 - p_0}{1 - p_1}} \\
h_1 &= \frac{\ln \frac{1 - \beta}{\alpha}}{\ln \frac{p_0}{p_1} + \ln \frac{1 - p_0}{1 - p_1}} \\
s &= \frac{\ln \frac{1 - p_0}{1 - p_1}}{\ln \frac{p_0}{p_1} + \ln \frac{1 - p_0}{1 - p_1}}
\end{align*}
\]

When \( \alpha = \beta \), \( h_0 = h_1 \).
Hence, in the $n \sim r$ plane, $n$ is abscissa and $r$ is ordinate, drawing two lines with slope $s$ and intercept $-h_0$, $h_1$ respectively.

$$L_0: r = -h_0 + sn$$
$$L_1: r = h_1 + sn$$

Let $n$ begin from 1, find the point $(n, r)$ one by one, and join them to a broken line. If the broken line goes below the beeline $L_0$, that is $r \leq -h_0 + sn$, judged products have qualified reliability and should be accepted. Thus $L_0$ is called qualification determination line. If the broken line goes up the beeline $L_1$, that is $r \geq h_1 + sn$, judged products have unqualified reliability and should be rejected. When $(n, r)$ is between $L_0$ and $L_1$, continuing test. Thus the area between $L_0$ and $L_1$ is called continuing test area.

Under normal case, in order to prevent $(n, r)$ from staying in the continuing test area, judgment can be given based on the relationship between $r$ and $r_n$ when $n = n_r$, where $n_r$ and $r_n$ are censored sample size and the number of censored failure.

III. SUCCESS RATIO INTEGRATED SEQUENTIAL SAMPLING PLAN MODEL

A. Basic Model

Assume that the development test data is $(n_r, r_n)$, where $n_r$ is the number of total tests or the equivalent number of total tests, and $r_n$ is the number of failure or the equivalent number of failure. Equivalent total test times and equivalent failure times are total test times and failure times which are gained through transformation using correlative method (like the method in Reference 4) when development test data does not follow binomial distribution.

Then, let $R$ is product’s reliability, which is obtained by the data from compliance test, and $R_0$ is product’s reliability which is obtained by development test data. The basic model using similarity coefficient $\alpha$ can be described as

$$f_R(x) \propto f_{R_0}^{\alpha}(x)$$

where $f_R(x)$ is reliability density function.

B. The Transformation of Development Test Data

Transform development test data firstly. Assuming that $R$ is product’s real reliability which is transformed from development test data, based on basic model, there is

$$f_R(x) \propto f_{R_0}^{\alpha}(x)$$

Performing standardization to $f_R(x)$, there is

$$f_R(x) = \frac{f_{R_0}^{\alpha}(x)}{\int f_{R_0}^{\alpha}(x)dx}$$

where $\int f_{R_0}^{\alpha}(x)dx$ is a constant. $f_R(x)$ is the reliability density function which is obtained by transformation from development test data.

C. Fusion of Test Data

The fusion of test data considers reliability distribution gained through transformation from development test data as prior distribution, using density function of Bayes formula, and integrates compliance test data $(n, r)$, where $n$ is total number of tests, and $r$ is the number of failures. Then posterior distribution $\pi(x|D)$ is given by

$$\pi(x|D) = \frac{p(D|x)\pi(x)}{\int_0^1 p(D|x)\pi(x)dx} = \frac{h(x, D)}{m(D)}$$

Where

$$h(x, D) = p(D = (n, r) | x) \ast f_R(x)$$

$$= \left( \frac{n_r}{n_r - r_n} \right)^{n_r - n} (1 - x)^{n_r - r_n} f_{R_0} \left( \frac{x}{1 - x}, \frac{1}{n_r - r_n} \right)$$

$$0 \leq x \leq 1$$

$$m(D) = \int_0^1 h(x, D)dx$$

$\pi(x|D)$ is the posterior distribution which can be obtained by the fusion of development test data and compliance test data. $\pi(x|D)$ is also product’s reliability posterior distribution density.

D. Acceptance and Rejection Judgement

According to reliability posterior distribution function, 10% quantile and 90% quantile of reliability can be given. Let them be $R_{0.1}$ and $R_{0.9}$. The conversion method is used.
\[
\left\{ \begin{array}{l}
\frac{1}{2} \int_0^{R_n'} \left[ \text{betapdf}(t, n' - f', f') + \text{betapdf}(t, n' - f', f' + 1) \right] dt = 0.1 \\
\frac{1}{2} \int_0^{R_n} \left[ \text{betapdf}(t, n' - f', f') + \text{betapdf}(t, n' - f', f' + 1) \right] dt = 0.9
\end{array} \right.
\]

(6)

Where \( \text{betapdf}(x, a, b) \) is Beta distribution density with parameters \( a \) and \( b \). \((n', f')\) can be obtained. \((n, f')\) represents equivalent test result which is gained by the fusion of development test data and compliance test data. \((n', f')\) can be drawn on the sequential sampling judgment graph, where producer’s quality, consumer’s quality and both sides of risk are given in advance. Then acceptance, rejection or going on test can be judged.

IV. APPLICATION OF SUCCESS RATIO INTEGRATED SEQUENTIAL SAMPLING PLAN

A. Application Condition

(1) There is some test data which has been gained from actual compliance test. Let \( n \geq \frac{h_0}{s} \), which means \( n \) must achieve the least test number which can be judged to an acceptance in standard sequential sampling plan.

(2) Sequential sampling plan using development test data should be regarded as non-censored sequential test. If actual test number achieves \( n \) sample size of single sampling, then the judgment can be given according to single sampling plan.

B. Application Steps

(1) Perform compliance test and obtain test data \((n, r)\), where \( n \geq \left[ \frac{h_0}{s} \right] \);

(2) Integrate development test data and field compliance test data to obtain posterior distribution \( \pi(x | D) \) after fusion;

(3) Convert posterior distribution into binomial data \((n_r, r)\);

\[
\alpha = \left\{ \begin{array}{l}
\int f_h(x) \log f_n(x) dx \\
\int f_h(x) \log f_n(x) dx
\end{array} \right. \leq \left\{ \begin{array}{l}
\int f_h(x) \log f_n(x) dx \\
\int f_h(x) \log f_n(x) dx
\end{array} \right.
\]

\[
\left\{ \begin{array}{l}
\int f_h(x) \log f_n(x) dx \\
\int f_h(x) \log f_n(x) dx
\end{array} \right. \geq \left\{ \begin{array}{l}
\int f_h(x) \log f_n(x) dx \\
\int f_h(x) \log f_n(x) dx
\end{array} \right.
\]

\[
\left\{ \begin{array}{l}
\int f_h(x) \log f_n(x) dx \\
\int f_h(x) \log f_n(x) dx
\end{array} \right.
\]

\[
\alpha \text{ is similarity degree between reliability distribution } F_h(x) \text{ and } F_n(x), \text{ where } f_h(x) \text{ is reliability density function.}
\]

(4) Carry out coordinate transformation to standard sequential plan.

Perform following change to qualified determination line and unqualified determination line where \( x = n_r \):

\[
\left\{ \begin{array}{l}
L_0 : r = h_0 + s n_r - r + r_n \\
L_1 : r = h_1 + s n_r - r + r_n
\end{array} \right.
\]

(7)

Coordinate transformation makes drawing point from test data more visual in engineering calculation and does not affect whole sampling plan.

(5) Give judgment

If \( r_n \leq L_0(n_r) \), products have qualified reliability and should be accept;

If \( r_n \geq L_1(n_r) \), products have unqualified reliability and should be reject;

If \( L_0(n_r) < r_n < L_1(n_r) \), products should continuing test. Return to step (1).

V. THE DETERMINATION OF SIMILARITY COEFFICIENT

The similarity coefficient in the above success ratio integrated sequential sampling plan is a measurement for similarity degree between test data and development test data. There are many methods to confirm similarity coefficient. This is a more reasonable method to confirm similarity coefficient, which gives similarity coefficient using the concept of differential entropy in Information Theory.

Let

\[
\alpha
\]

\[
\alpha \text{ is similarity degree between reliability distribution } F_h(x) \text{ and } F_n(x), \text{ where } f_h(x) \text{ is reliability density function.}
\]

VI. SIMULATION EXAMPLE

Carrying out reliability compliance test, regulation is given by both sides in advance: producer’s risk quality is \( p_0 = 0.1 \),
distinguishing ratio $D = 2$ (consumer’s risk quality $p_i = 0.2$), producer’s risk and consumer’s risk are both 20%. According to standard sequential sampling plan, they are:

$$s = 0.14524$$

$$h = 1.7095$$

$$n_r = 49, r_r = 7$$

Here, product’s real reliability is assumed to be 0.84. In practical sampling compliance test, product’s real reliability is unknown. This supposed value given here is to obtain simulative test sample sequence conveniently in this example, which is shown in TABLE I. Here, TT means Test Times and CST means Cumulative Success Times.

**TABLE I. SIMULATIVE TEST SAMPLE SEQUENCE**

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<tr>
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This product has performed development test in development phase. Assume that the conversion data obtained is (50, 7).

Standard sequential test graph and new integrated sequential test graph from the new method in this paper are shown as follows:

**REFERENCES**


