A Fuzzy Bayesian Reasoning Method to Realise Interactive Failure Analysis

Zaili Yang, Stephen Bonsall and Jin Wang
School of Engineering
Liverpool John Moores University
Liverpool, L3 3AF, UK
E-mails: [Z.Yang; S.Bonsall; J.Wang]@ljmu.ac.uk

Abstract—This paper presents a novel failure analysis and prioritization method, which takes the advantages of both fuzzy and Bayesian reasoning to deal with some of the drawbacks concerning the use of the conventional Failure Mode and Effects Analysis (FMEA) technique. The method is proposed to extend the previously developed Fuzzy Rule-based Bayesian Reasoning (FuRBaR) approach to a wider context, where not only ambiguous estimates but also dependent relations associated with failures can be appropriately modelled.

Keywords—FMEA, fuzzy-bayesian, maritime risk assessment.

I. INTRODUCTION

The justification of using the conventional Failure Mode and Effects Analysis (FMEA) technique to complex and large engineering systems has been the subject of considerable debate in recent years. It is due to the fact that modelling and analysing complex reliability problems increasingly requires to acquire precise failure data and to accurately simulate failure relations. However, in many realistic scenarios, failure analysis is often associated with uncertainty. Little numerical data of any statistical significance may be available to support traditional FMEA. The assumption of only having independent relations between failures in the FMEA may lead to wrong and costly safety decision in terms of complex system reliability analysis. Although many methods and methodologies [2], [4], [6], [8] have been developed to overcome the weaknesses exposed in the FMEA [7], [8], [1], it has been recognized that while few studies have been conducted to improve precise failure criticality analysis without compromising the advances of the classical US MIL-STD-1629A method, visibility and easiness, there are less to be taken into account with respect to the dependent relations between failures in the literature. Following the development of a Fuzzy Rule-based Bayesian Reasoning (FuRBaR) approach for carrying out failure criticality analysis as both precisely and visibly as possible [9], this paper presents a novel fuzzy Bayesian method, as the extension of the FuRBaR to realise interactive failure analysis. The kernel of the method includes two fuzzy Bayesian networks (BNs) and a fuzzy mapping link to connect them. One fuzzy BN uses Bayesian reasoning to infer fuzzy rule-based (FRB) FMEA (the FuRBaR), which is described in Section 2. The other network uses Bayesian reasoning to model the causal relations between failures. The fuzzy mapping link is based on the definition of failure likelihood fuzzy membership functions, which is introduced in Section 3. For a comparison purpose, the case study in [9] is used in Section 4 to demonstrate the superiority of the newly extended method over the standard FuRBaR. Section 5 concludes the paper with the presentation of its limitations and possible future work.

II. FURBAR APPROACH

The FuRBaR approach transforms belief degrees in rule bases into subjective conditional probabilities in BN. The transformation functions as an effective link make it possible to use the advances of both fuzzy and Bayesian inference. All necessary steps required for developing criticality analysis using the FuRBaR approach is briefly introduced as follows. More relevant details can be found from [9].

The first step is to construct FRB systems with belief structures in FMEA. Fuzzy IF-THEN rules with belief structures represent functional mappings between antecedents and conclusions, possibly with uncertainty. It provides a more informative, realistic scheme than a simple IF-THEN rule base on uncertain knowledge representation. An IF-THEN rule with belief structures can be expressed as follows:

\[ R_1; \text{IF} \ A_1^j, \ldots, A_n^j, \text{AND} \ A_n^j, \text{THEN} \ \{\beta_1^j, D_1\}, \ldots, \{\beta_N^j, D_N\} \ (\ N \ \sum_{j=1}^N \beta_j^j = 1) \]

where \( A_i^j \ (i \in M) \) indicates the linguistic variable of the \( j^{th} \) attribute used in the \( k^{th} \) rule, \( R_k; D_j \ (j \in N) \) means the \( j^{th} \) linguistic variables used in the conclusion part of the rule base; \( \beta_j^j \) represents the belief degree to which \( D_j \) is believed to be the consequence if in \( R^j \) the input satisfies the antecedent linguistic vector \( A_i^j \).

To construct such systems in FMEA, the three fundamental attributes, failure occurrence likelihood \( L \), consequence severity \( C \), and the chance of failure being undetected \( P \), are considered as the antecedent attributes in IF-THEN rules. Safety level \( S \) is expressed as the conclusion attribute. After the definition of all linguistic variables [9] used to describe \( L, C, P, \) and \( S \), the rule base can be constructed using the expert knowledge to define their relations and appropriately assign belief values \( \beta_i^j \). For example,

\[ R_1; \text{IF very low (L1)} \text{and negligible (C1)}, \text{AND reasonably unlikely (P1)}, \text{THEN} \ \{(1, \text{good (S1)}), (0, \text{average (S2)}), (0, \text{fair (S3)}), (0, \text{poor (S4)})\} \].
R$: IF very low (L1), and negligible (C1), and reasonably unlikely (P2), THEN \((0.91, \text{good (S1)}), (0.09, \text{average (S2)}), (0, \text{fair (S3)}), (0, \text{poor (S4)})\).

Such an IF-THEN rule base with belief structures can be presented in the form of conditional probabilities. For example, the second rule above can be displayed as follows.

Given \(L1, C1, \text{and } P2\), the probability of \(Sh = h = 1, \ldots, 4\) is \((0.91, 0.09, 0, 0)\) or

\[ p(Sh|L1, C1, P3) = (0.91, 0.09, 0, 0) \quad (h = 1, 2, 3, 4) \]

where “\(|\)” symbolizes conditional probability. Consequently, the entire rule base can be converted into and expressed by

\[ p(S|L, C, P) \]

Using a BN technique, the FRB constructed in FMEA can be modelled and transformed into a four-node converging connection. It includes three parent nodes, \(N_L, N_C\), and \(N_P\) (Nodes \(L, C\), and \(P\)); and one child node \(N_S\) (Node \(S\)). The states of each node are determined by the linguistics variables used to describe \(L, C\), and \(P\). From the work by [9], it can be seen that there are \(7\) states in \(N_L\), \(5\) in \(N_C\), \(7\) in \(N_P\), and \(4\) in \(N_S\). Having transferred the rule base into a BN framework, the rule-based risk inference for the failure criticality analysis will be simplified as the calculation of the marginal probability of the node \(N_S\). Having analysed \(p(Sh|Li, Cj, Pk) = h = 1, \ldots, 4, i = 1, \ldots, 7, j = 1, \ldots, 5, k = 1, \ldots, 7\), to marginalize \(S\), it requires the calculation of the unconditional prior probabilities of \(N_L\), \(N_C\) and \(N_P\), which are \(p(Li), p(Cj)\) and \(p(Pk)\), respectively.

Consequently, the next step is to estimate failure observation/input. If the failures are independent, then the failure input can be described by the linguistic variables associated with the antecedent attributes using various techniques such as the fuzzy Max-Min operation [4] and the subjective probability method [9]. Once \(p(Li), p(Cj)\) and \(p(Pk)\) are estimated, the safety level of each failure can be computed as follows:

\[
p(Sh) = \sum_{i=1}^{7} \sum_{j=1}^{5} \sum_{k=1}^{4} p(Sh|Li, Cj, Pk) p(Li) p(Cj) p(Pk) \quad (h = 1, \ldots, 4) \quad (1)
\]

Finally, to prioritize the failures, \(Sh = h = 1, \ldots, 4\) requires the assignment of appropriate utility values \(U_{Sh}\). The utility values can be defined on the basis of the combination of specific fuzzy rules [9]. Then a new failure priority/ranking index \((RI)\) can be developed as:

\[
RI = \sum_{h=1}^{4} p(Sh) U_{Sh} \quad (2)
\]

where the larger the value of \(RI\) is, the lower the risk level of potential failures.

III. A FUZZY BAYESIAN METHOD TO MODEL INTERACTIVE FAILURES

Like most of available methods in system reliability analysis (i.e. series/parallel configuration, tie-set and fault tree analysis), the traditional FMEA ignores the partial causal relations between failure modes. It is even worse that the technique fails to model the direct and indirect relations between risk factors [7]. Responding to the nature of system reliability problems, the most predictable relations among different failures may emerge at a variety of spatial, temporal or functional scales. Therefore, current safety knowledge might be better represented if each relation is described at or between the dynamic and interactive levels of detail at which the key failure events could be identified. Given this, the above assumption “if the failures are independent” may be arguable. There is therefore a need for new models of enabling the realization of interactive failure analysis.

BNs constitute a class of probabilistic models with strong connections to graph theory, which can be considered as a realistic way of structuring a situation for reasoning failure likelihood with an interactive feature. Recently, their popularity started to grow among system risk assessors and reliability analysts [3]. Common features in the precious studies could therefore be derived and applied to this investigation. If the failures are dependent, then the failure estimates, particularly the failure likelihood \(L\), need to be carefully processed before being used into the FurBaN using the framework as follows:

1. Identify failure modes.
2. Define the states of the failure modes as “working” and “failed”.
3. Analyse the casual relationships between the failures.
4. Construct the BN to qualitatively model the relationships.
5. Assign conditional probabilities of all failures to quantitatively model their relationships.
6. Calculate marginal (posterior) probabilities of the failures as updated failure likelihood.
7. Express the marginal probabilities using the linguistic variables, \(Li = i = 1, \ldots, 7\).

Apart from Steps 5 and 7, this framework is relatively straightforward. Regarding Step 5, a common criticism of the Bayesian approach is that it requires too much information in the form of prior probabilities, and that this information is often difficult or impossible to obtain in risk assessment. It may be beneficial for experts to use linguistic variables based on fuzzy sets rather than subjective probabilities based on point estimations to express their judgements and expertise to compensate the unavailability of historical failure data. As far as Step 7 is concerned, it is a common practice in risk assessment to convert probabilistic likelihood into linguistic variables by establishing their fuzzy membership functions on the likelihood plane. Obviously, the nature of the two steps is a basic fuzzification-defuzzification operation, which can be implemented by appropriately defining the fuzzy memberships of the seven linguistic variables in Table 1, as shown in Figure 1. The operation provides as an effective link to connect two BNs, where a centroid defuzzification approach [5] can be used to transform the linguistic estimate into crispy prior conditional probabilities and the updated posterior/marginal probabilities of each failure can be converted into and expressed by \(Li = i = 1, \ldots, 7\) with belief degrees.

IV. USING THE TEMPLATE

For comparison purposes, the case associated with the collision risk of a floating, production, storage, and offloading (FPSO) system and a shuttle tanker during tandem offloading operation in [9] is revisited. The four major causes to this technical failure are: controllable pitch propeller (CPP), thrusting (T), position reference system (PRS), and dynamic positioning system (DP) failures. From a perspective of system reliability analysis, the failure of PRS will affect the
operation of $DP$, which as a control system will probably further lead to the occurrence of $CPP$ and $T$ delivering wrong or low performance, respectively. This undoubtedly conflicts with the assumption of four failures being independent in the earlier work, which requires the reinvestigation of the failure likelihood estimates. For example, the previous estimate of failure, $CPP$'s occurrence likelihood ($0.738$ RF and $0.262$ F) in [9], can be converted into $p(CPP = \text{failed}) = 0.005$ ($= 0.738 * 10^{-2.57} + 0.262 * 10^{-1.57}$) using a centroid defuzzification approach. Consequently, $p(CPP = \text{working}) = 0.995$ can be obtained. Considering the relation between $CPP$ and $DP$, it is reasonable to judge $p(CPP = \text{failed} | DP = \text{working}) = 0.005$, $p(CPP = \text{working} | DP = \text{working}) = 0.995$, $p(CPP = \text{failed} | DP = \text{failed}) = 1$ and $p(CPP = \text{working} | DP = \text{failed}) = 0$. Similarly, $p(PRS)$, $p(DP | PRS)$ and $p(T | DP)$ can be calculated. With the presentation of all conditional probabilities, the $BN$ to model the quantitative relations between the failures is established and the marginal probability of each failure can be inferred and expressed as $p(PRS = \text{failed}) = 0.0036$, $p(DP = \text{failed}) = 0.0096$, $p(T = \text{failed}) = 0.0113$ and $p(CPP = \text{failed}) = 0.0144$. The necessity and significance of modelling interactive failures can be evidenced by the comparison of the prior and posterior probabilities of the failure $CPP$. Based on Figure 1, all the posterior probabilities can be transformed and expressed by $Li$ in order to infer the $RI$ values of the failures using the $FuRBaR$ approach. For example, $p(CPP = \text{failed}) = 0.0144$ ($= 10^{-3.45}$), which can be expressed by ($0.97$ F and $0.03$ HF). From [9], the $p(C)$ and $p(Pk)$ associated with $CPP$ are obtained; and the safety level and $RI$ of $CPP$ can be computed using the $FuRBaR$ approach as follows (in Figure 2).

$S(CPP) = 0 \text{ Good, 0 average, 0.8398 fair, 0.1602 poor} = 53.75$ (RI)

Similarly, the following $RI$ values of the other failures can be calculated:

$S(PRS) = 0 \text{ Good, 0.0189 average, 0.8491 fair, 0.1319 poor} = 60.84$ (RI)
$S(DP) = 0 \text{ Good, 0.251 average, 0.7855 fair, 0.1893 poor} = 58.89$ (RI)
$S(T) = 0 \text{ Good, 0.135 average, 0.7063 fair, 0.1902 poor} = 80.7$ (RI)

Consequently, the technical failure caused by malfunction of the $CPP$ systems requires more attention in the control of the collision risk compared to the other failure causes. In terms of risk contribution, the failure priority ordering is $CPP$, $DP$, $PRS$, and $T$, which is different with the ordering $DP$, $CPP$, $PRS$, and $T$ obtained using the standard $FuRBaR$ approach.

V. CONCLUSION

This paper has successfully demonstrated a new fuzzy Bayesian based interactive failure prioritization method that enhances the capability of the standard $FuRBaR$ in dealing with ambiguous estimates of failure modes by modelling their dependent relations. In order to optimize the extended $FuRBaR$ approach, future work requires the consideration of developing automatic computing software to integrate the two networks with their fuzzy link and the validation of the reliability of the fuzzy link in terms of avoiding information loss in the basic fuzzification-defuzzification conversion operation.

REFERENCES


<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Meaning (general interpretation)</th>
<th>Failure rate (1/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>Failure is unlikely but possible during lifetime</td>
<td>$&lt;10^{-6}$</td>
</tr>
<tr>
<td>Low (L)</td>
<td>Likely to happen once during lifetime</td>
<td>$0.25 \times 10^{-3}$</td>
</tr>
<tr>
<td>Reasonably low (RL)</td>
<td>Between low and average</td>
<td>$0.25 \times 10^{-4}$</td>
</tr>
<tr>
<td>Average (A)</td>
<td>Occasional failure</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Reasonably frequent (RF)</td>
<td>Likely to occur from time to time</td>
<td>$0.25 \times 10^{-4}$</td>
</tr>
<tr>
<td>Frequent (F)</td>
<td>Repeated failure</td>
<td>$0.125 \times 10^{-7}$</td>
</tr>
<tr>
<td>Highly frequent (HF)</td>
<td>Failure is almost inevitable or likely to happen repeatedly</td>
<td>$&gt;0.125 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

TABLE 1. LINGUISTIC VARIABLES USED TO DESCRIBE FAILURE LIKELIHOOD
Figure 1. Fuzzy set representation of the linguistic variables used to describe failure likelihood

Figure 2. CPP's safety level and RI value based on the extended FuRBaR method