A Mathematical Model of Equipment Maintainability Growth Based on Least Square Method

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Abstract—By analyzing actuality of maintainability, the meaning and process of maintainability growth are explained. Then a mathematical model of maintainability growth is given and the least square method is adopted to work out the parameters of the model. So a method that is used to assess present maintainability and calculate future maintainability is founded. Finally, with the maintainability growth test data, an example is given to further illustrate the rationality of the method.

Keywords—maintainability; maintainability growth; Least Square Method; mathematical model

I. INTRODUCTION

With the development of technology, the equipments of each country become increasingly electronic, digital, excellent and intelligent. Large and complicated equipment continually comes forth. On the one hand, the rapid improvement of equipment fighting performance makes the modern battle become unprecedented fury and cruelty, which leads to increase of damage probability. On the other hand, the durative exertion of equipment operational effectiveness more and more depends on equipment maintainability. The serious problems of maintainability caused by the increase of technique complexity mainly take on the following [1]:

- Life cycle cost increases obviously.
- Combat readiness descends distinctly.
- New equipment can’t come into being battle effectiveness

At present, the mass problems for equipment maintenance in our army are due to various reasons, but in the final analysis the low maintainability is one of the main reasons. The main bottleneck which restricts maintainability’s enhancement and improvement is being short of pertinent maintainability growth technology. It has great military meaning and obvious economic benefit for improving equipment maintainability and further improving army’s maintenance support ability to strengthen the research of equipment maintainability growth. Therefore, it is imperative to carry through a systematic research of equipment maintainability growth.

II. INTRODUCTION OF MAINTAINABILITY GROWTH

Maintainability isn’t invariable. And it should be improved continuously with the change of actual equipment maintenance demand, to achieve the goal of maintainability growth. By carrying on maintainability work designedly and adopting necessary corrective measures, the equipment’s failures in design and manufacture can be corrected step by step, so as to increase maintainability level continually. Equipment maintainability growth runs through the whole life cycle. It is an effective approach for solving problems of equipment maintenance to improve and enhance equipment maintainability to realize maintainability growth.

A. The Concept of Maintainability Growth

In the whole life cycle and in the repeating course of try-improve-retry, the defects of maintainability design and maintenance technique of equipment which limit maintainability level may emerge continuously. After analyzing, the equipment is improved gradually to be perfect. Consequently, the maintainability level of equipment is enhanced step by step, and this is maintainability growth [2]. The process that the product maintainability is gradually improved by continuously eliminating maintainability defects in design or manufacture is called maintainability growth process. In most cases, maintainability defects are found in tests or in use. Therefore, the process of maintainability growth can be described as a process of testing (using), analyzing and fixing (TAAF) [3].

B. The Phases of Maintainability Growth

Maintainability growth consists of two phases: maintainability growth in service phase and in development phase, as shown in Figure 1.

1) Maintainability growth in development phase

Maintainability growth can not come true without the efforts of development sector. In the early stages of development phase, the maintainability could not immediately reach the set target. It has to go through the process of testing-improving-retesting to increase maintainability. The design isn’t finalized in the early stages of development, so the maintainability growth can be achieved by the durative maintainability design based on the qualitative requirements of maintainability. Maintainability growth in service phase is fit to be applied on equipment which is designing. Maintainability is a set attribute when the equipment is designing. So after the design is finalized, the maintainability quality also is
determined. Therefore, the equipment must be redesigned to fundamentally improve its maintainability. However, it is bound to alter the structural intensity, component layout and so on, to redesign the shaped equipment. Therefore, maintainability design is not only the redesign of maintainability, but also the redesign of the entire equipment. Obviously, it is unrealistic and impossible to redesign the active-duty equipment.

In a word, maintainability mainly rests with products’ design, so the best time for maintainability growth is engineering development phase.

III. MATHEMATICAL MODEL

In the analysis and management of maintainability growth, some statistic methods may be used. The future maintainability level in the process of maintainability growth should be forecasted besides that the present maintainability level need be figured out. Therefore, in light of the characteristic of maintainability growth test plan, relevant mathematical models should be established so as to correctly track and forecast product maintainability and provide proper information for engineering management.

With the maintainability growth test going on, some related maintainability problems are found and solved continuously. Equipment maintainability level increases step by step and the trend of maintainability growth become milder and milder gradually. When maintainability level meets the set demand, the test can be stopped.

In this article, the MTTR is used as the parameter to scale maintainability level. The MTTR of a certain test phase \( t \) is marked as \( M(t) \). Suppose that the increase of \( M(t) \), which is marked as \( dM(t)/dt \), makes direct ratio with \( M(t) \) and inverse ratio with \( t^2 \), that is,

\[
dM(t)/dt = -k_2 M(t)/t^2
\]

The solution of the above differential equation is

\[
M(t) = k_1 e^{k_2/t}.
\]

If \( t' = 1/t \), then

\[
\ln M(t') = \ln k_1 + k_2 t'
\]

For a set of test values \( (M_i, t_i) (i=1,2,...,n) \), the corresponding \( (\ln M_i, t'_i) \) should be worked out. They satisfy the following equation.

\[
\ln M_i = \ln k_1 + k_2 t'_i + \epsilon_i
\]

The least square method will be used to calculate the estimators, which is marked as \( (\hat{k}_1, \hat{k}_2) \), of the unknown parameters \( (k_1, k_2) \), so as to make the following equation come true.
The estimator \((\hat{k}_1, \hat{k}_2)\) which satisfies the equation (2) is called as least squares estimator of \((k_1, k_2)\). It can be solved with the method of differential equation.

An expression is marked as

\[
Q(k_1, k_2) = \sum_{i=1}^{n} (\ln M_i - \ln k_1 - k_2 t_i')^2
\]

Making

\[
\frac{\partial Q}{\partial k_1} \bigg|_{(k_1, k_2)=(\hat{k}_1, \hat{k}_2)} = 0, \quad \frac{\partial Q}{\partial k_2} \bigg|_{(k_1, k_2)=(\hat{k}_1, \hat{k}_2)} = 0
\]

Then the equation (3) can finally show as follows:

\[
\begin{align*}
\hat{n} \hat{k}_1 + n \hat{t} \hat{k}_2 &= n \ln M \\
\hat{n} \hat{t} \hat{k}_1 + n \hat{t}^2 \hat{k}_2 &= \sum_{i=1}^{n} t_i' \ln M_i
\end{align*}
\]

Here,

\[
\hat{t} = \frac{1}{n} \sum_{i=1}^{n} t_i', \quad \ln M = \frac{1}{n} \sum_{i=1}^{n} \ln M_i
\]

Solve the equation (4) to get the estimator \((\hat{k}_1, \hat{k}_2)\) of the unknown parameters \((k_1, k_2)\) as follows:

\[
\hat{k}_1 = \ln M - \hat{k}_2 \hat{t}, \quad \hat{k}_2 = \frac{\sum_{i=1}^{n} (t_i' - \hat{t}) (\ln M_i - \ln M)}{\sum_{i=1}^{n} (t_i' - \hat{t})^2}
\]

IV. AN APPLIED EXAMPLE

The maintainability growth test cycle of a certain type of equipment was expected to have six phases. Some tests were done in each phase. The maintainability information of the former three phases was shown in the table 1. Some improvements were made to increase maintainability level after every test. Try to analyze how the equipment maintainability level will be after the six phases and how the maintainability that is estimated by the mathematical model is coincident with the actual maintainability.

Programming with the Matlab software, the least square method is adopted to work out the parameters as follows: \(k_1=1.2932, k_2=0.4349\). Then the equation (1) is used to calculate \(\hat{M}'\), as shown in the table 1.

<table>
<thead>
<tr>
<th>Test phases</th>
<th>(M_i)</th>
<th>(\hat{M}')</th>
<th>(\ln (M(t')))</th>
<th>(t')</th>
<th>(\hat{M}_i - M_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.9977</td>
<td>0.6931</td>
<td>1</td>
<td>-0.0023</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>1.6073</td>
<td>0.4700</td>
<td>0.5</td>
<td>0.0073</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1.4949</td>
<td>0.4055</td>
<td>0.5</td>
<td>-0.0051</td>
</tr>
<tr>
<td>4</td>
<td>1.4417</td>
<td>1.4107</td>
<td>0.3333</td>
<td>0.5</td>
<td>-0.0051</td>
</tr>
<tr>
<td>5</td>
<td>1.3904</td>
<td>1.3904</td>
<td>0.3333</td>
<td>0.5</td>
<td>0.0073</td>
</tr>
</tbody>
</table>

From Table 1, we can see that the absolute value of the difference between actual value \(M_i\) and estimated value \(\hat{M}_i\) is not more than 10^{-2}. Therefore, the mathematical model can approximatively assess the present maintainability and calculate the future maintainability. The mathematical model can be used for prediction of maintainability growth. Figure 1 is the curve of maintainability growth.

V. CONCLUSIONS

With the structure of new equipment becoming more and more complex, the maintenance problems have increasingly become the most intractable problems in the development of equipment. So a variety of mathematical models of maintainability growth must be set up as soon as possible. The maintainability growth can be controlled quantitatively by the mathematical models. With the uniform deployment of the manpower, outlay and time, the test, analysis and improvement of engineering are brought under scientific management so as
to avoid human and material waste caused by eyeless changes for equipment.

REFERENCES


