Evaluation of Military Aircraft Support Plan Based on Fuzzy Analytic Network Process in Development Phase

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Abstract—Evaluation of military aircraft support plan has a significant impact on the efficiency of aircraft system. In order to solve the problem that support plan evaluation has many defects in angles and methods in previous studies, the evaluation model based on fuzzy analytical network process (FANP) is presented. Firstly, support system characteristics are abstracted to determine evaluation elements (or clusters) of military aircraft support plan. Secondly, Analytical network process is used to describe relationships between two elements (or clusters) and compare their relative importance. Triangular fuzzy number is parameterized linguistic variables to handle with the vagueness and subjectivity of pairwise comparisons in a fuzzy environment. Two methods are synthesized to obtain final priority ranking of alternatives. Finally, evaluating certain type aircraft support plan is given as an example, and feasibility and validity of model are verified. It could provide an excellent decision supporting for defense industrial sectors to evaluate military aircraft support plans.

Keywords—evaluation; fuzzy analytical network process (FANP); military aircraft support plan

I. INTRODUCTION

Evaluating support plans can identify whether the support systems can satisfy the support requirements of materiel or not, and determine an optimal support plan. It makes the materiel and support system well coordinated, and has a significant impact on materiel system efficiency, so how to evaluate support plans is a vital important problem. In previous research, evaluation has some defects in angles and methods: (1) In the evaluation angles, most of the work has attempted to evaluate support resource elements or materiel system efficiency, but little work has analyzed general characteristics of support system and evaluated alternatives from the view of system characteristics, (2) In the selection of evaluation methods, data of support plans in development phase is not sufficient, so the method combining fuzzy comprehensive evaluation and analytical hierarchy process (AHP) is employed very widely for rational logic and straightforward computation processes. But, traditional AHP method simplifies a complex problem to a hierarchical structure where factors are assumed to be uncorrelated. Actually, there are complex interdependent relationships among factors of the support plan decision, and feedback that low-level factors affect high-level ones.

In order to solve the problems mentioned above, the paper abstracts general characteristics of support system to determine evaluation indexes of military aircraft support plans, and analytical network process (ANP) is applied to evaluate alternatives. This method put forward by Thomas L. Saaty in 1996 is a new theory that extends the AHP to cases of dependence and feedback among elements by a network structure, and remedies the limitation of the AHP. Considering decision-makers’ judgment about relative importance is subjective and ambiguities which cannot be captured by standard 9 point scaling, the paper uses triangular fuzzy number to parameterize linguistic variables to handle with the vagueness and subjectivity of pairwise comparisons. It is closer to actual making decision and more suitable to support plans evaluation and selection in development phase.

II. CHARACTERISTICS AND INDEXES OF MILITARY AIRCRAFT SUPPORT SYSTEM

A. Timeliness

Timeliness is a temporal characteristic of achieving support need. Timeliness consists of two parts: (1) support system can provide service immediately when materiel needs to be supported, i.e. timely commencement, (2) support system can complete rapidly whenever task starts to be conducted, i.e. timely completion. Timeliness of military aircraft support system is mainly evaluated by four elements: mean response time to operational support (MRTOS), mean response time to maintenance support (MRTMS), turnaround time (TAT) and mean maintenance hour per flight hour (MMH/FH). Response time to support is the time from a support need being produced to starting the support activities.

B. Efficiency

Efficiency is an efficient characteristic of achieving support need. Efficiency consists of two parts: (1) efficiency of support need being satisfied by support resource, i.e. fill rate, (2) efficiency of support resource being utilized, i.e. utilization rate. Efficiency of military aircraft support system is depicted with six elements: equipment utilization rate (EUR), facility utilization rate (FUR), air material utilization rate (AMUR), personnel utilization rate (PUR), equipment fill rate (EFR), facility fill rate (FFR), air material fill rate (AMFR) and personnel fill rate (PFR).
C. Deployability

Deployability is a capability of satisfying deployment need. It can reflect agility and flexibility of support system. It is closely related with the variety, quantity, volume and weight of the support resource to be deployed. Deployability of military aircraft support system is mainly evaluated by deployment footprint, which is the number of transportation used to transfer support resource. If the quantity and variety of support resource to be deployed and overall weight and volume of support resource packaged are certain, the number of vehicles required can be calculated by the load and volume of the vehicle.

D. Economy

Economy is the ability of achieving support system performance with the cost as little as possible. The assessment of economy of military aircraft support system mainly lies to mean operational cost per flight hour (MOC/FH) and mean maintenance cost per flight hour (MMC/FH).

As a result, evaluation indexes system of military aircraft support system is showed as Fig. 1.

Figure 1. Evaluation indexes system of military aircraft support system

III. EVALUATION OF MILITARY AIRCRAFT SUPPORT PLANS BASED ON FANP

The proposed method to evaluate military aircraft support plans based on FANP is illustrated in the following 5 steps.

A. Construct Support Plan Evaluation Network Analysis Structure

Relationships among evaluation indexes are analyzed as following: (1) the longer response time is, the longer waiting time for support resource and the lower fill rate of resource are, (2) the longer support time are, the longer working time of support resource and the bigger utilization rate of resource get, (3) numerous resources should be collocated to meet the demand of high fill rate, which would result in the reduction of utilization rate, (4) the smaller deployment footprint is, the less support resource and the lower the support cost need, which leads to the reduce fill rate of resource, enhance utilization rate of resource, and increase response time.

Four characteristics of support system and alternatives of support plans are called as clusters of elements and each characteristic index and alternative are elements in clusters. Interdependency relationships between two clusters and two elements are represented by two–way lines. Inner dependencies among the elements of a cluster are represented by one-way arcs. The network analysis model of military aircraft support plans is established as Fig. 2.

B. Construct Fuzzy Supermatrix

1) Conception and meaning of the triangular fuzzy number

Fuzzy set theory was introduced by Zadeh to deal with non-statistical information and data. It can mathematically express the relationships of vagueness and subjective. Among various membership functions, the triangular fuzzy number is the most popular in the engineering applications. A triangular fuzzy number is denoted simply by \( M(l, m, u) \), where \( l \leq m \leq u \). The parameters \( l \) and \( u \) respectively represent the smallest and the largest possible value, and \( m \) stands for the most promising value that describes a fuzzy event. \( M(l, m, u) \) has the following triangular type membership function.

\[
\mu_M(x) = \begin{cases} 
0 & x < l \text{ or } x > u \\
\frac{x-l}{m-l} & l \leq x \leq m \\
\frac{x-u}{m-u} & m \leq x \leq u 
\end{cases}
\]

A triangular fuzzy number can be shown in Fig. 3.

Figure 3. A triangular fuzzy number
It can be seen that $l$ and $u$ express fuzzy degree of judgment in a triangular fuzzy number. The more $u - l$ is, the higher fuzzy degree of judgment is. For $l = m = u$, $M$ is non-fuzzy number, which shows that judgment is not fuzzy. The linguistic variables for relative importance corresponding to triangular fuzzy numbers are shown in Tab. I.

<table>
<thead>
<tr>
<th>Triangular fuzzy number</th>
<th>Linguistic variables for relative importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/2, 1, 3/2)</td>
<td>The former and the latter are equally important</td>
</tr>
<tr>
<td>(5/2, 3, 7/2)</td>
<td>The former is moderately more important than the latter</td>
</tr>
<tr>
<td>(9/2, 5, 11/2)</td>
<td>The former is strongly more important than the latter</td>
</tr>
<tr>
<td>(13/2, 7, 15/2)</td>
<td>The former is very strongly more important than the latter</td>
</tr>
<tr>
<td>(17/2, 9, 19/2)</td>
<td>The former is extremely more important than the latter</td>
</tr>
</tbody>
</table>

Notes: ① (3/2, 2, 5/2), (11/2, 6, 13/2) and (15/2, 8, 17/2) denote intermediate values of the adjacent judgments above; ② If the importance ratio of element (cluster) $i$ and element (cluster) $j$ is $(l, m, u)$, the importance ratio of element (cluster) $j$ and element (cluster) $i$ is $(l, u, m) = (1/u, 1/m, 1/l)$

2) Conduct fuzzy judgment matrices of pairwise comparison and supermatrix

Assume that cluster $C_i$ has $n_i$ elements which are denoted as $e_{i1}, e_{i2}, \cdots, e_{in_i}$, and $C_j$ has $n_j$ elements which are denoted as $e_{j1}, e_{j2}, \cdots, e_{jn_j}$. Regarding $e_{ij}$ ($i = 1, 2, n_i$) as a criterion, the fuzzy judging matrix $B_i^j = (b_{pq}^j)_{n_i \times n_j}$ ($p, q = 1, 2, \cdots, n_j$) can be generated by comparing the elements of $C_j$ related to $e_{ij}$ with respect to the influence on $e_{ij}$, where all $b_{pq}^j$ are triangular fuzzy numbers, $b_{pq}^j = (l_{pq}^j, m_{pq}^j, u_{pq}^j)$. Considering that relative importance judgment between elements $e_{ip}$ and $e_{jq}$ ($p, q = 1, 2, \cdots, n_j$) made by the k-th expert is $b_{pq}^k$, the judgment matrix can be expressed as $B_i^j (k) = (b_{pq}^j)^k$. So $l_{pq}^j = \text{min}(b_{pq}^j)$, $m_{pq}^j = \text{Geomean}(b_{pq}^j)$, where Geomean denotes geometric average or weighted geometric average considering the priorities of experts, and $u_{pq}^j = \text{max}(b_{pq}^j)$. Fuzzy eigenvector of the fuzzy judgment matrix $B_i^j$ is denoted as $w_i^j = (w_i^1, w_i^2, \cdots, w_i^n)$. The fuzzy eigenvector needs to be defuzzied according to conception of cut set in fuzzy analysis [8-9]. Let

$$w_i^j(\alpha) = (w_i^{\text{max}} - w_i^{\text{min}})\alpha + w_i^{\text{max}}$$  \hspace{1cm} (2)

$$w_i^j(\alpha, \lambda) = \lambda w_i^{\text{max}}(\alpha) + (1 - \lambda)w_i^{\text{min}}(\alpha)$$  \hspace{1cm} (3)

Where $\alpha$ is the intercepting value coefficient, and $\alpha \in [0, 1]$. $\alpha$ reflects change degree of experts judgments. For $\alpha = 0$, it contains all weight information from experts. For $\alpha = 1$, it contains the least weight information from experts. $\lambda$ is the decision-making optimization coefficient, and $\lambda \in [0, 1]$. $\lambda$ reflects integration of decision-making. For $\lambda = 0$, all upper limit value is selected, which is optimal condition. For $\lambda = 1$, all lower limit value of weight is selected, which is the most conservative condition. In the real determination process, the higher consensus expert team has, the bigger $\alpha$ this model should select. The more optimism expert team has, the bigger $\lambda$ this model should select.

$W_i^j$ can be obtained by normalizing $w_i^j(\alpha, \lambda)$ and presented as (5):

$$w_i^j(\alpha, \lambda) = \frac{w_i^j(\alpha, \lambda)}{\sum_{i=1}^{n} w_i^j(\alpha, \lambda)}$$  \hspace{1cm} (4)

In the same way, regarding another element in cluster $C_i$ as a criterion, elements in cluster $C_j$ are compared to construct fuzzy judgment matrix of pairwise comparison. All fuzzy eigenvector of the fuzzy judgment matrices are normalized and presented as a supermatrix $W_i$. Considering the inner and outer relationship among all elements in all clusters, supermatrix $W_i$ is denoted as

$$W_i = \begin{bmatrix} C_1 \vdots & \vdots & \vdots & C_i \vdots & \vdots & \vdots & C_n \vdots \end{bmatrix}$$

$$\begin{bmatrix} e_{11} & \cdots & e_{14} & e_{12} & \cdots & e_{14} & e_{21} & \cdots & e_{24} & \cdots & e_{24} & e_{25} & \cdots & e_{25} & \cdots & e_{25} \\ W_{11} & \cdots & W_{14} & W_{12} & \cdots & W_{14} & W_{13} & \cdots & W_{14} & \cdots & W_{15} & \cdots & W_{15} & \cdots & W_{15} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_2 \vdots & \vdots & \vdots & C_3 \vdots & \vdots & \vdots & C_n \vdots \end{bmatrix}$$

$$w_i^j = \begin{bmatrix} w_i^{j1} & w_i^{j2} & \cdots & w_i^{jn_i} \\ w_i^{j1} & w_i^{j2} & \cdots & w_i^{jn_i} \\ \vdots & \vdots & \vdots & \vdots \\ w_i^{j1} & w_i^{j2} & \cdots & w_i^{jn_i} \end{bmatrix}$$

Where $W_i$
C. Calculate fuzzy weighted supermatrix

Regarding each element cluster as a criterion, fuzzy judgment matrices of pairwise comparing are established according to the method mentioned in the former section. Then corresponding eigenvectors are computed and normalized to obtain the weight matrix and represented as $A$. 

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

Each column of supermatrix $W$ is weighted by the corresponding weights of $A$, and result is known as the weighted supermatrix $W^\alpha$. 

$$W^\alpha = AW$$  \hspace{1cm} (9)

D. Calculate Limit supermatrix

Interdependency relationship among the factors makes the computation of elements priorities complex. For example, $W$ can reflect direct comparison relationship between $i$th element and $j$th element, and $\sum_{i \neq j} W_{ij}W_{ji}$ can be used to represent indirect comparison relationship between them, and iterative supermatrix represents complex indirect comparison relationship. Therefore, limit supermatrix should be computed to obtain steady priorities. The limit supermatrix has the same form as the weighted supermatrix, but all the columns of it are the same. 

$$W^\alpha = \lim_{k \to \infty} W^\alpha_k$$  \hspace{1cm} (10)

E. Synthesize the priority of alternatives

By normalizing elements in each element clusters based on the result of (10), priority vectors of alternatives and weights of characteristics evaluation indexes are acquirable.

IV. CASE STUDY

There are three support plans of military aircraft to be evaluated, and we need select an optimal alternative. Evaluation indexes system and network analysis structure are shown as Fig.1 and Fig.2. For example, under the criterion “mean response time to operational support”, equipment fill rate, facility fill rate, air material fill rate and personnel fill rate are relative with the criterion in the efficiency indexes. Fuzzy judgment matrix of pairwise comparison is constructed, and eigenvector is calculated by choosing $\alpha = 0$ and $\lambda = 1$ in Tab. II.

| TABLE II. THE FUZZY JUDGMENT MATRIX OF RELATIVE IMPORTANCE OF EFFICIENCY INDEXES ABOUT MRT |  
|-----------------------------------------------|-----------------------------------------------|
| **MRT** | **EFR** | **FFR** | **AMFR** | **PFR** |
|-----------------------------------------------|-----------------------------------------------|
| (1/2,1/3,2) | (3/2,2/5,2) | (7/2,13/3,2) | (1/2,1,3,2) |       |
| (2/5,1/2,2/3) | (1/2,1,3,2) | (2/9,14/2,7) | (1/2,1,3,2) |       |
| (5/2,3,7/2) | (7/2,4,9/2) | (1/2,1,3,2) | (5/2,3,7/2) |       |
| (2/3,1/2) | (3/2,2/5,2) | (2/7,13/3,2) | (1/2,1,3,2) |       |
| **Eigenvector** | 0.1894 | 0.1054 | 0.5158 | 0.1894 |

Priority vectors of the three alternatives are obtained by normalizing 0.1236, 0.0422, 0.0594, and shown as Fig. 4. It’s obvious that the ranking of alternatives is alternative 1, alternative 3, and alternative 2.

![Image](381x151 to 485x234)

Figure 4. The priority vectors of alternatives
Weights of evaluation indexes are listed as Tab. III.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Evaluation index</th>
<th>Weight</th>
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</thead>
<tbody>
<tr>
<td>Timeliness</td>
<td>MRT₆₆</td>
<td>0.3636</td>
</tr>
<tr>
<td></td>
<td>MRT₆₈</td>
<td>0.3858</td>
</tr>
<tr>
<td></td>
<td>TAT</td>
<td>0.1367</td>
</tr>
<tr>
<td></td>
<td>MMH/FH</td>
<td>0.1139</td>
</tr>
<tr>
<td>Efficiency</td>
<td>EUR</td>
<td>0.0728</td>
</tr>
<tr>
<td></td>
<td>FUR</td>
<td>0.0400</td>
</tr>
<tr>
<td></td>
<td>AMUR</td>
<td>0.1281</td>
</tr>
<tr>
<td></td>
<td>PUR</td>
<td>0.1122</td>
</tr>
<tr>
<td></td>
<td>EFR</td>
<td>0.1240</td>
</tr>
<tr>
<td></td>
<td>TFR</td>
<td>0.0691</td>
</tr>
<tr>
<td></td>
<td>AMFR</td>
<td>0.3196</td>
</tr>
<tr>
<td></td>
<td>PFR</td>
<td>0.1342</td>
</tr>
<tr>
<td>Deployability</td>
<td>DP</td>
<td>1</td>
</tr>
<tr>
<td>Economy</td>
<td>MOC/FH</td>
<td>0.4181</td>
</tr>
<tr>
<td></td>
<td>MMC/FH</td>
<td>0.5819</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

This paper evaluates military aircraft support plan from the view of general characteristics by applying FANP, which is closer to actual making decision than AHP and can improve the precision of alternatives priority and make the selection of alternatives more scientific. Evaluating certain type aircraft support plans is given as an example, feasibility and validity of model was verified. It could provide an excellent method for evaluating military aircraft support plans.

REFERENCES