An Adaptive Threshold Based on Support Vector Machine for Fault Diagnosis

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Abstract—Considering the drawback of the big error when using fixed threshold in fault diagnosis for hydraulic servo system, many factors that may affect the fault threshold are analyzed. By integrating the key factors in threshold model, such as modeling error, random disturbance, input instructions, system current status and etc, an adaptive threshold scheme for fault diagnosis is proposed in this paper, which is based on a pattern recognition algorithm called support vector machine (SVM). It is very effective to adaptively adjust the fault threshold according to a variety of influencing factors. And the robustness is improved by the proposed method, which is verified by experimental results.

Keywords- Adaptive threshold; Support vector machine (SVM); fault diagnosis; hydraulic servo system; actuator

I. INTRODUCTION

The residual error between system output and model output can be used in model-based fault diagnosis. If the residual error is close to zero, system is considered to be in an operational condition; else if the residual error is greater than zero, system is considered to be in fault. But the residual error is not ideally zero in practice, which is caused by the existing noise, random disturbance and the difference between model and system. Thus, it can not be used as the rule for fault diagnosis whether the residual error is zero or not. So a proper threshold is introduced into model-based fault diagnosis. And the rule is described as following: if the residual error is greater than the threshold, system is considered to be in fault; while other cases, system is considered to be in an operational condition.

The threshold will directly affect the fault diagnosis performance. Too large threshold may result in missing detection; on the contrary, it may cause false alarm. Furthermore, the residual error may vary with different conditions of the key factors, such as modeling error, random disturbance, input instructions, system current status and etc. So the threshold needs to be adaptively adjusted. Thus, it has become a valuable topic to select the proper adaptive threshold.

II. ANALYSIS OF INFLUENCING FACTORS OF RESIDUAL ERROR

Because the residual error varying directly influences the fault threshold, the key factors should be analyzed from the perspective of mathematical modeling, which is of the most importance in model-based fault diagnosis. Next, system model will be used as an object to analyze the various factors of the residual error.

According to the system modeling error and some other factors like disturbance and noise, the general equation needs to add a disturbance item, which is described as follows,

$$
\dot{x}_d = \begin{bmatrix} \dot{x} \\ \dot{x}_d \\ \end{bmatrix} = \begin{bmatrix} A + \Delta A & A_{10} \\ A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x(t) \\ x_d(t) \end{bmatrix} + \begin{bmatrix} B + \Delta B \\ B_d \end{bmatrix} u(t) + \begin{bmatrix} E_0 \\ E_d \end{bmatrix} m(t) + \begin{bmatrix} G \\ G_d \end{bmatrix} f(t)
$$

(1)
\[
y(t) = [C \quad 0] \begin{bmatrix} x(t) \\ x_y(t) \end{bmatrix} + Du(t) + Qf(t)
= Cx(t) + Du(t) + Qf_y(t)
\]  

(2)

where, \( x_n \in \mathbb{R}^N \), is the actual state variable, and \( N \) is the order of the dynamic system; \( x(t) \in \mathbb{R}^n \), is the adaptive state variable for system mathematical modeling, and \( n \) is the order of the system model; \( u(t) \in \mathbb{R}^r \), \( y(t) \in \mathbb{R}^m \), are the input vector and the output vector respectively; \( n(t) \in \mathbb{R}^r \), is the noise vector; \( f_y(t) \in \mathbb{R}^r \), is the actuator fault vector; \( f_q(t) \in \mathbb{R}^r \), is the sensor fault vector; \( (A,B,C,D) \) is the parameter matrix of the system model; \( A_{uo}, A_{y}, A_{u2}, \Delta A, B \) and \( \Delta B \) represent the systematic uncertainty and parametric variation; \( E_s \) and \( E_d \) are the system noise allocation matrix; \( G \) and \( G_s \) are the actuator fault allocation matrix; \( Q \) is the sensor fault allocation matrix. Since \( (A,B,C,D) \) represents the simplified system model, the following equations can be easily deduced,

\[
\begin{align*}
\dot{x} &= Ax(t) + Bu(t) + Ed(t) + Gf_y(t) \\
y(t) &= Cx(t) + Du(t) + Qf_y(t)
\end{align*}
\]  

(3)

(4)

where \( E = [\Delta A \quad \Delta B \quad E_s \quad A_{uo}] \), \( d(t) = [x(t), u(t), n(t), x_y(t)] \)

\( d(t) \in \mathbb{R}^l \), describes the uncertain input, such as parameter variation, modeling error, noise and disturbance etc. \( E \in \mathbb{R}^{m \times n} \) is the uncertain input allocation matrix.

For the actual system described above, select generation mode of the residual error based on state observer, which can be used to analyze the influencing factors.

The state equations of the observer are expressed as,

\[
\begin{align*}
\dot{x} &= Ax(t) + Bu(t) + H[y(t) - \hat{y}(t)] \\
\hat{y}(t) &= C\hat{x}(t) + Du(t)
\end{align*}
\]  

(5)

(6)

where, \( \hat{x} \in \mathbb{R}^n \), is the state estimation vector; \( \hat{y}(t) \in \mathbb{R}^m \), is the output estimation vector; \( H \) is the observer gain matrix.

Error equations of the system can be written as

\[
\epsilon(t) = (A - HC)e(t) + Ed(t) + Gf_y(t) - HQf_y(t)
\]  

(7)

\[
\epsilon(t) = Ce(t) + Qf_y(t)
\]  

(8)

where, \( e(t) = x(t) - \hat{x}(t) \), is the state estimation error; \( \epsilon(t) = y(t) - \hat{y}(t) \), is the output estimation error. According to the result of Laplace transform of equation (7), equation (8) can be written as:

\[
\epsilon(s) = C[sI - (A - HC)]^{-1}e(t_o) + C[sI - (A - HC)]^{-1}Ed(s) + C[sI - (A - HC)]^{-1}Gf_y(s) - C[sI - (A - HC)]^{-1}HQf_y(s) + Qf_y(s)
\]  

(9)

where \( e(t_o) \) is initial value of the state estimation error. The equation (9) is defined as the threshold. It is obvious that the residual error and system original state are related to the factors including system input, system uncertainty (such as parameter variation, modeling error, noise, disturbance and etc.), sensor fault and actuator fault, which should be comprehensively taken into account to calculate the adaptive threshold.

III. ADAPTIVE THRESHOLDING BASED ON SVM FOR FAULT DIAGNOSIS

A. Adaptive Threshold Model Derivation Taking the Residual Error into Account

From the analysis above, the adaptive threshold is related to the main factors including system input, output, disturbance and parameters drifting over time, which are taken into account for non-linear system adaptive threshold modeling. Let \( \epsilon(s) \) be the Laplace transform of the threshold, \( G(s) \) be transfer function of the actual system, \( G_n(s) \) be transfer function of the model, \( R(s) \) be the Laplace transform of the system input; and \( d \) represents the disturbance, \( \Delta(s) \) is the vector composed of drifting parameter, \( [R(d)](s) \) represents the Laplace transform of system input subject to disturbance, while \( [Y(d)](s) \) represents the Laplace transform of system output under disturbance. And the threshold model is

\[
\epsilon(s) = [[G(s, \Delta(s))[s] - G_n(s)]\times[R(d)](s)
\]  

(10)

The equation (10) can be written as

\[
\epsilon(s) = F[d, \Delta(s), R(s), Y(s)]
\]  

(11)

where, \( F \) is the mapping function of threshold, which takes a variety of influencing factors in frequency-domain into account. Because disturbance and parameter drifting are difficult to measure in practice, they are usually ignored when calculating the threshold. On that condition, we can get

\[
\epsilon(s) = F[R(s), Y(s)].
\]  

(12)

But the system output \( Y(s) \) is unknown in fault. So the equation (12) can not be used in practice. Because output of the model is very close to that of the normal system, when parallel model of the system has already been identified, the output of the model \( Y_n(s) \) can be used to calculate the adaptive threshold instead of \( Y(s) \). That can be written as

\[
\epsilon(s) = F[R(s), Y_n(s)].
\]  

(13)
B. The Adaptive Threshold Based on SVM

For the fitting process of non-linear mapping $F$, some people use polynomial fitting method which required that input data must be included in the training sample, or else the output is unpredictable. At the same time, polynomial is not fit to high dimensional fitting because of its complexity and unsteadiness. Neural network can be used to identify the non-linear mapping $F$ in high dimensional space, however, the generalization ability of neural network is poor. $F$ can be identified by SVM from the equation (13), as long as we know the system input, model output and the residual error. SVM can be used to identify any non-linear function, and it has very good generalization ability. So the SVM method is adopted to calculate the adaptive threshold in this paper. Compared with the neural network, this method can achieve the same precision with much less training samples. The adaptive threshold method based on SVM is shown in figure 1.

![The adaptive threshold method based on SVM](image)

The adaptive threshold can be easily acquired by bringing the system input and the model output into the SVM for calculation.

C. Identification algorithm of SVM

Compared with neural network, SVM has strict theoretical system and firm mathematical foundation without the local minimum problem. Even though sample size is small, the generalization ability of SVM is still good. That means the dependence on sample size is weak. Hence, SVM has been applied in many fields, such as pattern recognition, regression estimate, data mining and control theory.

The specific identification algorithm is as follows. $\{x_i, y_i\}$ the sample set of input and output for identified system is given. where $i = 1, ..., l$; $x_i \in R^n$ is the input vector; while $y_i \in R$ is the output vector. The identification of SVM is determining the corresponding output $y$ in a certain precision scope for any input $x$ by identifying a function $f$ through data training. That means SVM uses a non-linear mapping function $\phi(*)$ to map the sample to higher dimensional Eigen space. In this way, the non-linear function estimation is transformed into linear function estimation in higher dimensional Eigen space. That can be described by

$$f(x) = w^T \phi(x) + b$$

(14)

where, $\|w\| \leq constant$. Introducing slack variable $\xi$ and $\xi'$ into the equation (14), resolving the parameters of the equation (14) is equivalent to resolving the following optimization problem.

The object function is

$$\min_{w, b, \xi, \xi'} \frac{1}{2} w^T w + C \sum_{i=1}^{l} (\xi_i + \xi'_i)$$

(15)

and it must satisfy the following condition,

$$\begin{align*}
    y_i - w^T \phi(x_i) - b &\leq \varepsilon_i + \xi_i, \\
    w^T \phi(x_i) + b - y_i &\leq \varepsilon_i + \xi'_i, \\
    \xi_i, \xi'_i &\geq 0, i = 1, ..., l
\end{align*}$$

(16)

where the constant $C (C > 0)$ represents the cost of control training error, or called “punishment parameter”, which is usually used in balancing the rule item and the experience error, representing the smoothness of the function $f = \phi$ and $\xi'$ are the error introduced into the training set; $\varepsilon$ representing the permitted loss of training, turns the support vector into the sparse subset of training data. That can be described as

$$|y - f(x)| = \begin{cases} 
    0 & |y - f(x)| \leq \varepsilon \\
    |y - f(x)| - \varepsilon & others
\end{cases}$$

(17)

Through resolving the optimization problem of (15), the parameters of (14) can be easily derived. And the system identification is carried out.

IV. CASE STUDY OF ADAPTIVE THRESHOLDING IN FAULT DIAGNOSIS BASED ON SVM FOR HYDRAULIC SERVO SYSTEM

We analyze the actuator of normal state, and investigate the model identification precision and changes of adaptive threshold and that of the residual error by modifying the PID parameters and actuator quality. Figure 2 shows the curve of adaptive threshold when the actuator is in normal state. Figure 3 shows the curve of adaptive threshold when the actuator is in fault. Figure 4 shows the curve of adaptive threshold when the controller is in fault. Figure 5 shows the curve of adaptive threshold when the hydraulic actuating cylinder is in fault.

![Figure 1. The adaptive threshold method based on SVM](image)
Adaptive fault threshold when the actuator is of the normal state.

Adaptive fault threshold when the controller is in fault.

Adaptive fault threshold when the actuator is in fault.

Adaptive threshold when the hydraulic actuating cylinder is in fault.

### TABLE I. THE COMPARISON OF THE FIXED THRESHOLD METHOD AND THE ADAPTIVE THRESHOLD METHOD BY FAULT DIAGNOSIS RATE

<table>
<thead>
<tr>
<th>The selected method of threshold</th>
<th>Fault detection rate</th>
<th>False alarm rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional fixed threshold method</td>
<td>90%</td>
<td>9%</td>
</tr>
<tr>
<td>Adaptive threshold method</td>
<td>97%</td>
<td>4%</td>
</tr>
</tbody>
</table>

Obviously, adaptive threshold is not only reflecting the changes of the system signal with the characteristic of anti-disturbance in normal state, but is also used for fast and efficient fault diagnosis. Compared with the fixed threshold method, this method can effectively improve the accuracy of fault detection and restrain false alarm. That is shown in table 1.

### V. CONCLUSIONS

Aimed at existing problems when using fixed threshold in fault diagnosis for hydraulic servo system, influence factors of the fault threshold are comprehensively analyzed in this paper. An adaptive threshold scheme for fault diagnosis is proposed, which is based on SVM considering of modeling error, random disturbance, input instructions and system current status. The proposed method is verified by experimental results of actuator’s three states including normal state, control parameter variation state and actuator weight varying state. Through the comparison of detection results between adaptive threshold and fixed threshold, it indicates that the adaptive threshold method for fault diagnosis is very effective to restrain false alarm and improve the accuracy of fault detection.

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