Research on the Reliability Analysis Method of an Actuator Under Stochastic Load

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Abstract—Traditional time-dependent reliability analysis methods aim to predict the product failure time based on binary component and system status, which have not considered the influence of some stochastic characters for the reliability such as internal parameters variation and environment factors fluctuation. Current integrated design method considers performance and reliability requirements simultaneously, and has synthesized all the uncertain factors during the reliability analysis process, while this method is based on Monte-Carlo and large number of simulation times, which delivers low efficiency extremely. As we have known, system’s unreliability is always resulted from all kinds of random factors, so this paper presents a special reliability analysis method for a flight control actuator system under stochastic processes. First, we introduce the concept of the stochastic processes briefly, then we give an expression model for the stochastic wind, based on the mathematical model and load model, and suppose all the uncertainties come from the stochastic wind, we analyze the effect of the stochastic load for the actuator rotation angle. Furthermore, we select some performance characters as the criteria and calculate the actuator system reliability under the stochastic load. Finally, we create the mathematical model and stochastic wind model in Matlab, and compare the reliability value getting through simulation and analysis method, result shows that this method is feasible.

Keywords- Reliability; Stochastic Processes;Actuator; Wind Turbulence

I. INTRODUCTION

As we have known, system’s unreliability is always resulted from all kinds of random factors, which can be defined as stable stochastic processes based on their historic statistic data. If we get the mathematical description of the stochastic input of a system, the statistic regulation of the system output can be given through some analysis. For a specified system, we can get performance reliability value through its criterion based system output response under stochastic input, which presents a high efficient reliability calculation tool for current research about integration design.

At the beginning of this research, we describe the concept of the stochastic processes briefly, then we select an electrical actuator as the case, and create the mathematical model and stochastic load model of the actuator based the wind turbulence model. Suppose all the uncertainties come from the stochastic wind, we present the actuator’s reliability calculation method based its performance characters, through which we can get the reliability much more efficiently. Finally, we do some simulation based the actuator model, load model and wind turbulence model, and compare the analysis result with the simulation data, comparing result shows that this method is feasible. This paper is organized as follows: Section 2 introduces some basic theory about stochastic and the reliability calculation method of a system under the stochastic input; Section 3 presents the actuator model, wind turbulence model, load model, and gives a reliability value of the actuator under the stochastic load result from the wind turbulence. Section 4 gives a simulation model for the actuator, and gets another reliability value through 10000 simulation times, then we compare the results through two different methods, which can indicate this method is feasible. Section 5 presents the conclusion and some future work about this research.

II. RESPONSE OF LTI SYSTEM UNDER STABLE STOCHASTIC PROCESSES

A. The Concept

In the general Linear Time Invariant (LTI) system, input signal is always an exact function with the variable of time t, but the input, especially the noise of most real system can not be described accurately. Generally, we define this kind of random processes as the stochastic processes or stochastic functions, which can be expressed using some statistic
characters such as the mean value and standard variance. During most unstable stochastic processes, these statistic characters vary when time changing. There is not sophisticated technology to describe the unstable processes currently, in this paper, we suppose the stochastic load come from the wind turbulence is a stable stochastic process.

Let $x(t)$ be a stable stochastic process, the expectation of the random variable $x$ at time $t$ is defined by the means of an integral:

$$\bar{x} = \int_{-\infty}^{\infty} x f(x) dx$$

(1)

Where, $f(x)$ is the one order probability density function;

Variance of the random variable $x$ at time $t$ is:

$$\sigma^2 = (x - \bar{x})^2 = x^2 - \bar{x}^2$$

(2)

$R(\tau)$ is defined as the related function of the stochastic process $x(t)$.

$$R(\tau) = x(t)x(t+\tau)$$

(3)

$$R(0) = \sigma^2, \quad R(-\tau) = R(\tau), \quad \lim_{\tau \to \infty} R(\tau) = 0$$

(4)

Power density spectrum (PDS) $\Phi(\omega)$ is an image of the related function $R(\tau)$, which can be defined as the Fourier transform of $R(\tau)$.

$$\Phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega \tau} d\tau$$

(5)

Contrarily, $R(\tau)$ is defined as the inverse Fourier transform of $\Phi(\omega)$.

For a stable stochastic process $x(t)$, if $\bar{x} = 0$, based the equation (2)(3)(4)(6), we can get:

$$\sigma^2 = R(0) = 2\int_{0}^{\infty} \Phi(\omega) d\omega$$

(6)

For a single input single output (SISO) LTI system, PDS of the output signal $\Phi_y(\omega)$ can be given through the equation:

$$\Phi_y(\omega) = |F(i\omega)|^2 \Phi_x(\omega)$$

(8)

$\Phi_x(\omega)$: PDS of the input vector;

$F(i\omega)$: Transfer Function in frequency domain.

B. Probability distribution of the random variable
For a random variable $Y$, we can get its probability through Chebyshev’s inequality:

$$P\left[\left|Y - \bar{Y}\right| \geq \epsilon\right] \leq \frac{\sigma^2}{\epsilon^2}$$

(9)

$\bar{Y}$: mean value, $\sigma^2$: variance, $\epsilon$: any positive value;

But in fact, Chebyshev’s inequality gives estimation with too much margin. In most application, the random variable $Y(t)$ can be supposed to follow the Gauss distribution approximately, so we can get the Gauss inequality [8]:

$$P\left[\left|Y - \bar{Y}\right| \geq k\sigma\right] \equiv \frac{e^{-\frac{\epsilon^2}{2\sigma^2}}}{k\sqrt{2\pi}}$$

(10)

III. MODEL OF THE ACTUATOR AND WIND TURBULENCE
A. Actuator
The actuator incorporates a controller, a DC Motor drives the gearbox, surface, angle sensor and its amplifier. The stochastic load comes from the wind turbulence, which impacts the system by the means of joint torque. Fig. 1 shows the actuator scheme.

![Figure 1. Actuator Scheme](image)

Neglecting the non-linear factors, we can get the actuator model as shown in Fig. 2.

At a certain work condition, the joint torque model can be linearized, we can get the transfer function between the input
\( R \), \( V_{\text{wind}} \) and the output \( \theta \) shown in equations (11) and (12).

\[
F(s) = \frac{\Theta(s)}{R(s)} = \frac{K_{s}K_{e}}{L_{c}J_{ns} + (L_{c}K_{s} + R_{c}J_{ns} + K_{s}K_{e})}\]  
\[
F_{z}(s) = \frac{\Theta(s)}{V_{z}(s)} = \frac{K_{s}(L_{c}J_{ns} + R_{c}J_{ns} + K_{s}K_{e})}{L_{c}J_{ns} + (L_{c}K_{s} + K_{s}K_{e})}\]  

Random variable \( V_{\text{wind}} \) is a stochastic process, equation (12) shows that system’s output angle \( \theta \) is also a stochastic process by the infection of the wind, purpose of this research is analyzing the statistic characters of the output under the stochastic load, and calculating the actuator reliability by the specified performance criteria.

**B. Wind Turbulence Model**

Select a coordinate \( O_{1}x_{1}x_{2}x_{3} \), and the corresponding velocity vector \( (u_{1}, u_{2}, u_{3}) \), then we get the related function \( R_{y}(\xi) \) of the \( u_{i} \) (velocity at the position \( r \)) and the \( u_{l} \) (velocity at the position \( r + \xi \))

\[
R_{y}(\xi) = u(r)u_{l}(r + \xi) = \lim_{r \to + \infty} \frac{1}{2l} \int_{r}^{r + l} u(r)u_{l}(r + \xi)dr\]

Base the assumption about the wind turbulence, \( R_{y}(\xi) \) can be expressed as [9]:

\[
R_{y}(\xi, \xi_{1}, \xi_{2}) = \sigma^{2} \left\{ f(\xi) - g(\xi) \right\} \frac{\xi}{\xi} + g(\xi) \delta_{y}\]

where, \( \delta_{y} \) is the Kronecker operator;

\[
\delta_{y} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}, \quad \xi^{2} = \xi_{1}^{2} + \xi_{2}^{2} + \xi_{3}^{2}\]

\( \sigma^{2} \): Variance of the wind velocity;

\( f(\xi) \) is the longitudinal related function; \( g(\xi) \) is the lateral related function;

\[
g(\xi) = f(\xi) - \frac{1}{2} \xi \frac{df(\xi)}{d\xi}\]

In the Dryden Wind Turbulence Model, the longitudinal and lateral related functions are:

\[
f(\xi) = e^{-\xi^{2} / 2}, \quad g(\xi) = f(\xi) \left( 1 - \frac{\xi}{2L_{c}} \right)\]

In the Dryden Wind Turbulence Model, the PDS of \( u_{i} \) is:

\[
\Phi_{u_{i}}(\omega) = \Phi_{u_{i}}(\omega) = \sigma_{u}^{2} \frac{L_{c}}{\pi V_{\text{earth}}} \frac{1}{1 + (L_{c} \omega/V_{\text{earth}})^{2}}\]

where,

\[
L_{c} = \frac{h}{(0.177 + 0.000823h)^{1/2}}, \quad \sigma_{u} = \frac{0.1 \times u_{y0}}{(0.177 + 0.000823h)^{1/2}}\]

**C. Power density spectrum of the actuator response under the stochastic wind turbulence**

Equation (17) shows the PDS of the wind turbulence, which is the input signal in the actuator system, and the output signal is the angle \( \theta \). We can get the PDS of the \( \theta \) through equation (8)(12)(17).

\[
\sigma_{\theta}^{2} = 2 \int_{0}^{1} \Phi_{\theta}(\omega) d\omega = 9.4282 \times 10^{-7} \text{ (rad}^{2})\]

\[
\sigma_{\theta} = 9.7 \times 10^{-3} \text{ (rad) } = 0.0556\]

**D. Reliability analysis of the actuator under the stochastic wind turbulence**

Failure criteria of the actuator: output angle \( \theta \) is proportional with the input signal, if the actuator can not response with the input proportionally, we define that it is fail. Base the above criteria, we can define the failure condition is that the fluctuation range of \( \theta \) exceeding a threshold value within a constant input value. Supposed the failure threshold of the \( \theta \) is 0.2 degree, and all the fluctuation come from the wind turbulence, we can get the reliability calculation equation based equation (10) and (20):

\[
R = 1 - P \left\{ \theta \geq k \sigma_{\theta} \right\} \approx 1 - e^{-\frac{k \sigma_{\theta}}{2^{3/2}}} \approx 0.9998\]

Table 1 lists all the parameters used in this paper.
TABLE I.  PARAMETERS LIST

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value(Unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Gain of the controller</td>
<td>380(NULL)</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Armature inductance</td>
<td>0.3(mH)</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Armature resistance</td>
<td>0.8(Ω)</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Torque constant</td>
<td>0.044(N.m/A)</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Back EMF constant</td>
<td>0.045V/(rad/s)</td>
</tr>
<tr>
<td>$J$</td>
<td>Motor inertia</td>
<td>2.59e-5(N.m.s^2)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Damper rating</td>
<td>1.2e-4(N.m/(rad/s))</td>
</tr>
<tr>
<td>$n$</td>
<td>Reducer rate</td>
<td>300(NULL)</td>
</tr>
<tr>
<td>$L_{w}$</td>
<td>Wind turbulence scale length.</td>
<td>768(m)</td>
</tr>
<tr>
<td>$u_{20}$</td>
<td>Wind speed at 20 feet altitude.</td>
<td>3(m/s)</td>
</tr>
<tr>
<td>$K_f$</td>
<td>Gain of the sensor</td>
<td>0.5(NULL)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
<td>Variable(Kg/m^3)</td>
</tr>
<tr>
<td>$V_{wind}$</td>
<td>Wind speed</td>
<td>Variable (m/s)</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of the aircraft.</td>
<td>230(m)</td>
</tr>
<tr>
<td>$V_{earth}$</td>
<td>Aircraft velocity</td>
<td>246(m/s)</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Average chord length</td>
<td>0.064(m^2)</td>
</tr>
<tr>
<td>$C_{th}$</td>
<td>Joint torque coefficient</td>
<td>0.005(NULL)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Actuator rotation angle</td>
<td>10(°)</td>
</tr>
<tr>
<td>$\sigma_{w}^2$</td>
<td>Wind turbulence variance</td>
<td>0.35(m/s)</td>
</tr>
</tbody>
</table>

IV. SIMULATION MODEL AND RESULT

Actuator simulation model is created in Matlab, which includes the controller, wind turbulence, joint torque, DC motor, reducer and sensor, as shown in Fig. 3.

Base the mission profile of the aircraft, we set the simulation time is 60 seconds, and the input is a step rotation angle signal of 10°. After 10000 simulation loops, we have analyzed the fluctuation range of $\theta$ when it come to the stable status. Simulation result is shown in table 2.

TABLE II.  STATISTICS OF SIMULATION RESULT

<table>
<thead>
<tr>
<th>Fluctuation range of $\theta$ (°)</th>
<th>Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,0.05)</td>
<td>8356</td>
</tr>
<tr>
<td>[0.05,0.1)</td>
<td>1263</td>
</tr>
<tr>
<td>[0.1,0.15)</td>
<td>326</td>
</tr>
<tr>
<td>[0.15,0.2)</td>
<td>41</td>
</tr>
<tr>
<td>[0.2,∞)</td>
<td>14</td>
</tr>
</tbody>
</table>

Base the simulation results shown in table 2 and the failure criteria mentioned in section 3.4, we can get the reliability under the stochastic wind turbulence.

$$R = 1 - \frac{14}{10000} = 0.9986$$ (22)

Actuator mathematical model described in section 3 has some little difference with the simulation model shown in figure 3, which has neglected all the non-linear factors in real system. This difference result that the reliability values getting through equation (21) and (22) are not equal absolutely, but we think that this difference can not disprove this method’s dependability.

V. CONCLUSIONS

Product unreliability is always resulted from all kinds of random factors, response of a system under stochastic processes can be achieved through analyzing the PDS of random input signal and the system; furthermore, the reliability can be given based on the criteria of the response. This research selects a flight control actuator as the case, and describes the stochastic wind turbulence firstly. Then the statistic character of the actuator rotation angle was analyzed. Finally, we get the probability of unreliability result from the stochastic wind. Equation (21) and (22) give the reliability value through analysis method and the simulation showing that this method is feasible.

While this research only selects the wind turbulence as the stochastic process, and research result can only be used to evaluate the reliability when the stochastic process is stable, in most condition, this method can not be used because of the following reasons: firstly, system reliability is always effected by multi unstable stochastic process; secondly, the research system is not the ideal LTI system generally; finally, during the analyzing process, we select the performance characters as the criteria of failure, which results in that only selecting one threshold value as the criteria is not reasonable. All these condition require us to do some further research in the following aspects;

1) Expression method of the unstable stochastic processes;
2) Analyzing method of system response when the system is affected by multi stochastic processes;
3) Multi-states reliability evaluating method;
4) PDS calculation method of the unlinear system.

REFERENCES


