State Tree Analysis of FOG Based on Drift Brownian Motion

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Abstract—The binary state is the fundamental assumption to the traditional FTA. Hence, the traditional FTA gives only the binary logical analysis of product catastrophic failures. However, performance of Fiber Optical Gyroscope (FOG) will degrade with time and it is able to perform its task with partial performance. To overcome this problem, multistate reliability analysis is presented. However, multistate reliability analysis is conducted by assuming that products have a number of discrete states. In most cases, product physical parameters are continuous real variables. Consequently, we combine the state analysis to FTA, and construct the state tree of FOG. By assuming that the state of SLD can be depicted by the drift Brownian Motion (DBM), we calculate the top event probability. Finally, we contrast the two top event probabilities of DBM and the model in [1], respectively. (Abstract)

Keywords—state analysis; fault tree analysis; performance degradation; FOG (key words)

I. INTRODUCTION

Fault Tree Analysis (FTA) is commonly used as a method to reliability analysis. The binary state is the fundamental assumption to the traditional FTA, that is, the failure criterion of product is “all or nothing”. Hence, the traditional FTA gives only the binary logical analysis of product catastrophic failures. However, performance of some product will degrade with the time and such product is able to perform their task with partial performance. If product random failure process is simplified into a random variable (time to failure), and the relationship among reliability, failure mechanism and physical parameters is totally ignored, the reliability improvement action and strategy will be affected.

To overcome this problem, multistate reliability analysis is presented and there are many theoretical results and applications of multistate reliability analysis [1][2][3]. Multistate reliability analysis is conducted by assuming that products have a number of discrete states. Similarly, this assumption is also accompanied with some mathematical calculation and engineering application problems. In most cases, product physical parameters are continuous real variables. With the assumption that product’s performance is determined by its physical parameters, continuous state analysis is proposed. Consequently, product reliability models can be based on product’s physical parameters.

II. STATE TREE OF FOG

A. The traditional FTA of FOG

Generally, FOG is composed of optical part, circuit part, mechanical part and some configured software. Based on the importance of these parts, this paper will conduct FTA on optical and circuit part. According to the results from [3], the traditional fault tree of FOG is shown in Figure 1 and its event description in Table 1.

Fig. 1. the traditional fault tree of FOG

Fiber Optical Gyroscope (FOG) is a sensor which is based around the Sagnac Effect discovered in 1917 by a physicist called Georges Sagnac. A FOG is used to measure instantaneously the rotational speed of a mobile platform. A FOG provides extremely precise rotational rate information, in part because of its lack of cross-axis sensitivity to vibration, acceleration, and shock. Unlike the classic spinning-mass gyroscope, the FOG has virtually no moving parts and no inertial resistance to movement. Hence FOG technology is considered as one of the most reliable gyroscope technologies. Because of their great merits, FOG is used in high performance space, surveying, stabilization and inertial navigation tasks.

Since the performance of FOG will degrade with time, e.g. bias stability, we introduce the state tree analysis (STA) to help us improve the FOG reliability from design, manufacturing and operating.
During the operation of FOG, the performance of some optical components will descend because of the accumulative damage caused by environment stresses. And FOG will be greatly affected by the performance degradation of its critical components \cite{1}. From \cite{5}, \cite{7} and \cite{3}, the additional error caused by the circuit flaw will make the optical power of SLD directly relevant to the bias of FOG. Consequently, the FOG will manifest the change of the bias when the optical component performance reduction. Therefore, the traditional FTA cannot satisfy the FOG.

**B. The state tree analysis of FOG**

According to the analysis of section 2.1 and \cite{1}, we proposed the following assumptions to STA.

**A1** The basic events are independent

**A2** The optical power of the SLD will degrade with time, and its state space is denoted as the real interval \([0, M]\), while the states space of the other components of FOG are \([0, M]\)

**A3** Except for the SLD, the other components of FOG follow exponential distribution.

Based on the above assumptions and analysis, the state tree of FOG is shown in Figure 2, and its event description in Table 2.

### Table I. The Events of FOG FTA

<table>
<thead>
<tr>
<th>No.</th>
<th>Event Description</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Failure of FOG</td>
<td>Top</td>
</tr>
<tr>
<td>E₁</td>
<td>Failure of SLD</td>
<td></td>
</tr>
<tr>
<td>E₂</td>
<td>Failure of Y Waveguide</td>
<td></td>
</tr>
<tr>
<td>E₃</td>
<td>Failure of signal detection circuit</td>
<td></td>
</tr>
<tr>
<td>E₄</td>
<td>Failure of Polarization maintaining fiber couplers</td>
<td></td>
</tr>
<tr>
<td>E₅</td>
<td>Failure of SLD controller</td>
<td></td>
</tr>
<tr>
<td>E₆</td>
<td>Failure of fiber ring</td>
<td></td>
</tr>
<tr>
<td>E₇</td>
<td>Failure of photoelectric detector module (PIN-FET)</td>
<td></td>
</tr>
</tbody>
</table>

### Table II. The Events of FOG STA

<table>
<thead>
<tr>
<th>No.</th>
<th>Event Description</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>y₁</td>
<td>The state of FOG</td>
<td>Top</td>
</tr>
<tr>
<td>x₂</td>
<td>State of SLD</td>
<td></td>
</tr>
<tr>
<td>x₃</td>
<td>State of Y Waveguide</td>
<td></td>
</tr>
<tr>
<td>x₄</td>
<td>State of Polarization maintaining fiber couplers</td>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>x₆</td>
<td>State of signal detection circuit</td>
<td></td>
</tr>
<tr>
<td>x₇</td>
<td>State of SLD controller</td>
<td></td>
</tr>
</tbody>
</table>

**III. The Quantitative analysis**

**A. The probability of the top event**

For the catastrophic failure shown in Figure 2, if we assume that the reliability function of the event \(y₂\) is \(R_C(t)\), and the cdf. of the basic events \(x₂ \sim x₇\), are \(F_{x₂}(t)\sim F_{x₇}(t)\), then

\[
R_C(t) = 1 - \prod_{i=2}^{7} F_{x_i}(t) \tag{1}
\]

For the given critical value \(D\) of the SLD performance, if we assume that the reliability function of the event \(y₁\), i.e. \(x₁\), is \(R_D(t)\), then we can obtain the probability of the top event:

\[
F(t) = 1 - R(t) = 1 - R_C(t)R_D(t) \tag{2}
\]
B. The analysis of the basic events

The probabilities of the catastrophic basic events come from [3] (see Table 3).

<table>
<thead>
<tr>
<th>Event</th>
<th>Event Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$</td>
<td>0.0043</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.0072</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.0063</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.0005</td>
</tr>
<tr>
<td>$x_6$</td>
<td>0.00055</td>
</tr>
<tr>
<td>$x_7$</td>
<td>0.00047</td>
</tr>
</tbody>
</table>

Based on the above results and combined with equation (1), we can obtain the reliability of the medium event $y_2$, $R_C(10^4)=0.9807$.

Brownian motion is one of the most powerful stochastic processes in continuous time and continuous space. Also, Brownian motion is an essential ingredient in stochastic calculus and is basic for defining one of the most important classes of Markov processes, the diffusion processes, and solving large sample estimation problems in mathematical statistics. Thus, Brownian motion has fruitful applications in disciplines as physics, economics, communication theory and reliability theory.

This paper will utilize the Brownian motion to describe the performance degradation process and compare with the model used by [3].

If we assume that the optical power degradation of the SLD follows the linear Drift Brownian Motion (DBM), its reliability function and the cumulative probability function can be written as the equation (3) and equation (4) respectively based on the first passage time:

$$R(t) = \Phi\left(\frac{a - y_0 - \mu t}{\sigma \sqrt{t}}\right) - \Phi\left(\frac{a - y_0 + \mu t}{\sigma \sqrt{t}}\right)$$

$$F(t) = \Phi\left(\frac{\mu t}{\sigma \sqrt{t}}\right) + \Phi\left(\frac{-a + y_0}{\sigma \sqrt{t}}\right)$$

Where, $a$ is the threshold of the performance, $\mu$ is the drift parameter, $y_0$ is the initial degradation level at time $t_0$, $\sigma$ is the dispersion parameter.

There are the two unknown parameters in the equation (3), $\sigma$ and $\mu$. And by the maximum likelihood method, we can obtain their estimation and the equation (5) is the estimation function:

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \Delta Y_j}{m(n-1)\Delta t}, \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} (\Delta Y_j - \hat{\mu} \Delta t)^2}{m(n-1)\Delta t}}$$

Where, $\Delta Y_j$ is the degradation increment, $\Delta t$ is the measure interval, $m$ and $n$ are the cumulative inspection number and the sample size respectively.

Armed with the analysis in section 2.1, the additional error caused by the circuit flaw will make the optical power of SLD directly relevant to the bias of FOG. In order to collect the degradation data of the SLD’s optical power, we conduct the periodical inspection on twenty SLDs under 25 ºC. And the inspection interval is 48 hours. The degradation data is shown in Figure 3.

With the equation (5), $\hat{\mu} = -0.0029, \hat{\sigma} = 0.2765$. If the degradation threshold is the 50% of the initial optical power, we can get the reliability curve of the SLD as shown in Figure 4. When $t=10^4$hrs, $R_D(10^4)=0.9369$. Consequently, the top event probability is,

$$F(10^4h) = 1 - R(10^4h) = 1 - R_C(10^4h)R_D(10^4h) = 0.0812$$

![Figure 3. The optical power of SLDs](image)

![Figure 4. The SLD reliability curve (DBM)](image)

In the way of [1], we can assume that the degradation process is a Gaussian process. If $Y(t)$ is the degradation process, then $Y(t)$ is a Gaussian process with the mean $\tau(t)$ and variance $\xi(t)$. Similarly, if $D$ denote the critical value of the performance degradation, the reliability function is:

$$R(t) = \Phi\{Y(t) > D\} = \Phi\left[\frac{D - \tau(t)}{\xi(t)}\right]$$

Based on the above degradation data and the way of [1], $\hat{\tau}(t) = -8.63 \times 10^{-7} t, \hat{\xi}(t) = 8.7 \times 10^{-7} t + 1.2$. Hence, if the threshold
of the SLD optical power is the half of the initial, we can get the reliability curve as shown in Figure 5. and when $t=10^4 \text{hrs}$, $R'(10^4\text{hrs})=0.8341$.

Similarly, the top event probability calculated by the way of [1] is

$$F'(10^4\text{h})=1-R'(10^4\text{h})=1-R_c(10^4\text{h})R'(10^4\text{h})=0.182$$

![Figure 5. the SLD reliability curve ([1])](image)

IV. COMPARISON AND ANALYSIS

This paper deals with the state tree analysis by the use of the two degradation fitting ways. Their analysis results are shown in the Table 4.

<table>
<thead>
<tr>
<th>Method</th>
<th>DBM</th>
<th>Reference [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(10^4\text{h})$</td>
<td>0.0812</td>
<td>0.182</td>
</tr>
</tbody>
</table>

From the results of STA as shown in the Table 4, it is obviously that the top event probabilities considered degradation are both greater than the one without degradation in the same time. It is because:

1) Difference between estimation methods

In [1], the author firstly calculated the values of $r(t)$ and $\xi(t)$ at observation time with these 20 SLDs. However, this step will cut out much random and deterministic information carried by samples. Consequently, the final results estimated by the succeeding data fitting are not accurate.

For the DBM, it makes sufficient and direct use of degrading information in all samples to estimate drift parameter $\mu$ and dispersion parameter $\sigma$ by Maximum likelihood method. Hence, it improves the precision of the estimation.

2) Differences between reliability model

a) Fitting model for variance

Although the two models, the DBM and the model in [1], are Gaussian processes, the variance in [1] is the function with regard to $t$ while the relationship between the variance and time in the DBM is linear. For the DBM, the rigorous physical background and mathematical deduction make it more sense. In comparison, the model in [1] is constituted by some limited samples and it seems to lack some theoretical support.

b) Different modeling perspective

For the Gaussian process used in [1], its reliability model comes from the probability expression $\Pr\{Y(t)>D\}$. And this expression can be written as:

$$\Pr\{Y(t)>D\} = \Pr\{Y(t)>D|T_D \leq t\} \Pr\{T_D \leq t\} + \Pr\{Y(t)>D|T_D > t\} \Pr\{T_D > t\}$$  \hspace{1cm} (7)

From equation (7), $\Pr\{Y(t)>D\}$ denotes the probability that the random degradation process does not across the threshold. However, the reliability model for the DBM i.e. equation (3) is based on the first passage time. From the perspective of probability theory, this indicates a condition that if $T_a$ is the time when the random degradation process $Y(t)$ firstly crosses threshold $a$, then $Y(t)$ never across $a$. Hence, the probability expression based on the DBM first passage time is:

$$\Pr\left\{ \sup_{0 \leq t \leq T_a} Y(t) \geq D \right\}$$  \hspace{1cm} (8)

ACKNOWLEDGMENT

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